estec
European Space Research
and Technology Centre
Keplerlaan 1
2201 AZ Noordwijk
The Netherlands
T +31 (0)71 565 6565
F +31 (0)71 565 6040
www.esa.int

DOCUMENT

Accuracy of EOCFI Software functions in the Lib and Orbit libraries

Prepared by	**M. De Bartolomei (EOP-PES)**
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APPROVAL

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CHANGE LOG

Reason for change	Issue	Revision	Date
First Issue	1	0	27/10/2014
Completed with accuracy determination of functions not covered in the previous issue of this document.	2	0	3/11/2015
Updated reference to latest EOCFI SW version. Changed status to issued.	2	1	13/2/2017

CHANGE RECORD

Issue 2	**Revision** 0		
Reason for change	**Date**	**Pages**	**Paragraph(s)**
First Issue	27/10/2014	All	All
Following sections updated: - from 3.1.1 to 3.1.5: interpolators and propagators: added velocity accuracy determination. Following sections added: - 2.3: Co-ordinate system conversion; - 2.4: Geodesic distance computation; - 2.5: Geoid computation; - 3.2: Conversion from OSV to TLE; - 3.3: Conversion of position on orbit (OPS angle) to UTC time. Updated accordingly Introduction (section 1) and Annex A.	3/11/2015	All	All
Updated reference to latest EOCFI SW version. Changed status to issued.	13/2/2017	4	1

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1 INTRODUCTION

This note presents the accuracy of computations performed using the Earth Observation Mission CFI Software functions (or shortly EOCFI, see [RD01] and [RD02] for more details) in the Lib (see [RD03]) and Orbit (see [RD04]) libraries. Functions belonging to other libraries such as Pointing and Visibility will be covered by a dedicated document.
All Lib and Orbit functions are listed in Table 16 and Table 17 (see Annex A). For several of them (highlighted in grey), accuracy determination is not relevant. Functions for which accuracy determination is required are highlighted in green when they are presented in this version of this document.

With reference to Table 16 and Table 17, for each function the following details are provided:

- Name, Description and Outputs;
- Algorithm Type, that can be:
    - Model: the computation is based on a model;
    - Iterative: the computation is done iteratively;
    - Analytical: the computation is done using closed form expressions;
    - EOCFI specific: the method used has been specifically developed for the EOCFI SW;
    - N/A: Not Applicable (e.g. it is an initialisation function).
- Accuracy Determination, that can be:
    - N/A: Not Applicable (when algorithm type is N/A);
    - Not Required: when the algorithm is analytical, no accuracy determination is required. The correctness of analytical computations is verified at Software testing level, e.g. validating the reversibility of conversions;
    - Required: accuracy determination is required when the algorithm is CFI specific, iterative or model based.
- Document Reference: the document in which the accuracy determination is presented.

All computations described in this document have been performed using a MacBook Pro Computer (2.4 GHz Intel Core i7 Processor, 8 GB 1333 MHz DDR3 Memory).

References

Id	Title
[RD01]	EOCFI documentation main page, http://eop-cfi.esa.int/index.php/mission-cfi-software/eocfi-software/branch-4-x/eocfi-v4x-documentation
[RD02]	"Earth Observation Mission CFI Software - General Software User Manual", v4.12, http://eop-cfi.esa.int/REPO/PUBLIC/DOCUMENTATION/CFI/EOCFI/BRANCH_4X/4.12/C-Docs/SUMs/GeneralSUM_v4_12.pdf
[RD03]	"Earth Observation Mission CFI Software - LIB Software User Manual", v4.12, http://eop-cfi.esa.int/REPO/PUBLIC/DOCUMENTATION/CFI/EOCFI/BRANCH_4X/4.12/C-Docs/SUMs/LibSUM_v4_12.pdf
[RD04]	"Earth Observation Mission CFI Software – ORBIT General Software User Manual", v4.12, http://eop-cfi.esa.int/REPO/PUBLIC/DOCUMENTATION/CFI/EOCFI/BRANCH_4X/4.12/C-Docs/SUMs/OrbitSUM_v4_12.pdf
[RD05]	van Flandern, T. C.; Pulkkinen, K. F., "Low-precision formulae for planetary positions " http://adsabs.harvard.edu/abs/1979ApJS...41..391V
[RD06]	Bowring, B. R. (1976). "Transformation from Spatial to Geographical Coordinates" Surv. Rev. 23 (181): 323–327
[RD07]	IERS Bulletins, http://ww2.iers.org/IERS/EN/Publications/Bulletins/bulletins.html
[RD08]	TLE documentation at NORAD, https://celestrak.com/NORAD/documentation/
[RD09]	Standards of Fundamental Astronomy, http://www.iausofa.org/
[RD10]	GeographicLib, http://geographiclib.sourceforge.net/
[RD11]	NGA/NASA EGM96, http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm96/egm96.html

2 LIB LIBRARY

2.1 Computation of Sun, Moon, Planets positions

Functions **xl_sun, xl_moon and xl_planet** use formulae developed by Flandern et al. (see [RD05]). As stated by the authors, the accuracy of such methods is **1 arcminute (~0.017 deg).**

2.2 Conversion from Cartesian to Geodetic Co-ordinates

xl_cart_to_geod uses the Browing method (see [RD06]). The longitude computation is error-free as it is done using an analytical method. Latitude and altitude are computed using an iterative algorithm, therefore the accuracy depends on the threshold at which the iteration stops.

In order to determine the accuracy, the following steps have been executed: a point of a given longitude, latitude and altitude has been converted to Earth Fixed co-ordinates using function **xl_geod_to_cart** (this conversion is error-free at is it is done using an analytical method); such Earth Fixed co-ordinates have been converted back to longitude, latitude and altitude using **xl_cart_to_geod**; the result of this last computation has been compared with the original values of latitude and altitude. The sequence has been repeated at various latitudes and altitudes. Figure 1 shows the difference between computed values (latitude and altitudes) and expected ones as function of latitude. The maximum differences are shown in Table 1.

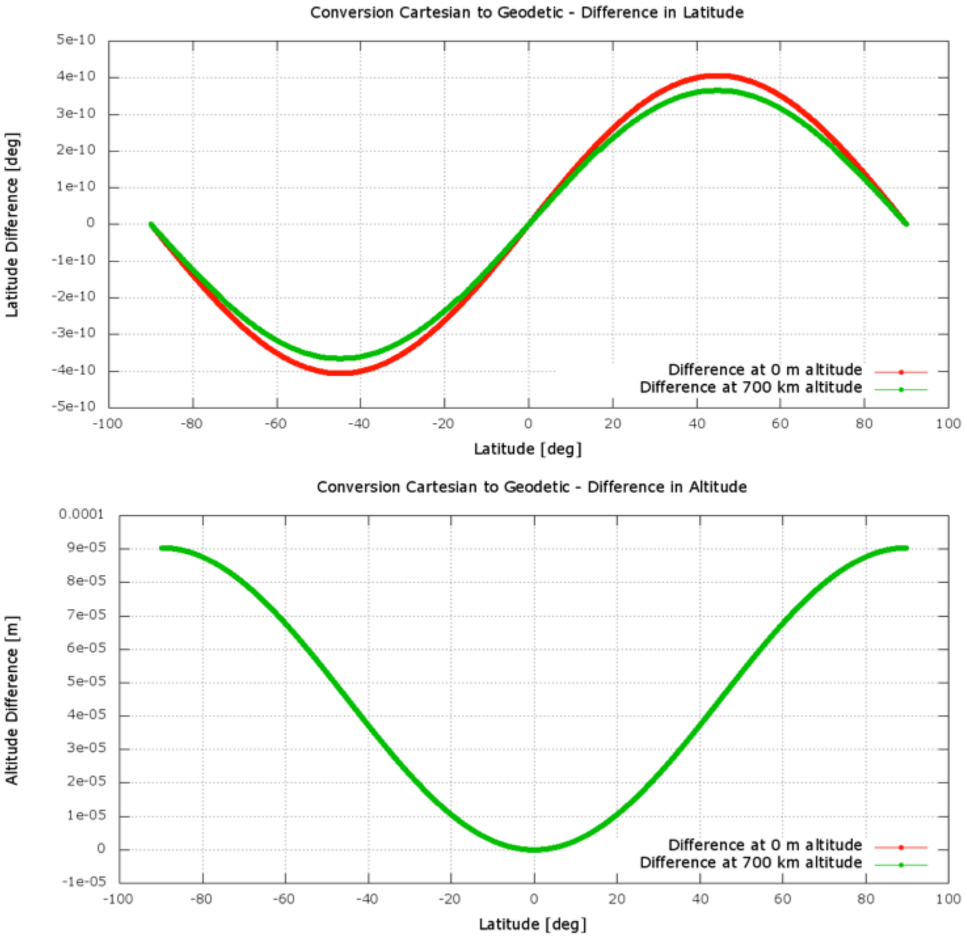


Figure 1 - Difference between computed and expected latitude and altitude

Reference Altitude	Max. Latitude Difference [deg]	Max. Altitude Difference [m]
0 m (Earth Surface)	4.0e-10	9.0e-5
700 km (Satellite height)	3.6e-10	9.0e-5

Table 1 - Maximum difference between expected and computed latitude and altitude

2.3 Co-ordinate system conversion

xl_change_cart_cs converts a vector from one co-ordinate system to another (e.g. from Earth Fixed to True of Date). The accuracy of this function has been determined by comparing its output to the one of the SOFA Software library. The SOFA (Standards of Fundamental Astronomy, see [RD09]) library is an accessible and authoritative set of algorithms and procedures that implement standard models used in fundamental astronomy.

The following steps have been executed: a list of OSVs in the Earth Fixed co-ordinate system has been generated (by propagation) along one orbit; such OSVs have been converted from Earth Fixed to ToD and GM2000 using both **xl_change_cart_cs** and the SOFA library; the same test has been repeated using the same OSVs but at different times of the year.

The difference between vectors computed using **xl_change_cart_cs** and EOCFI is shown in Figure 2. It can be seen that the difference is:

- 1.4 m Maximum for conversion from Earth Fixed to GM2000;
- 2.1 m Maximum for conversion from Earth Fixed to ToD.

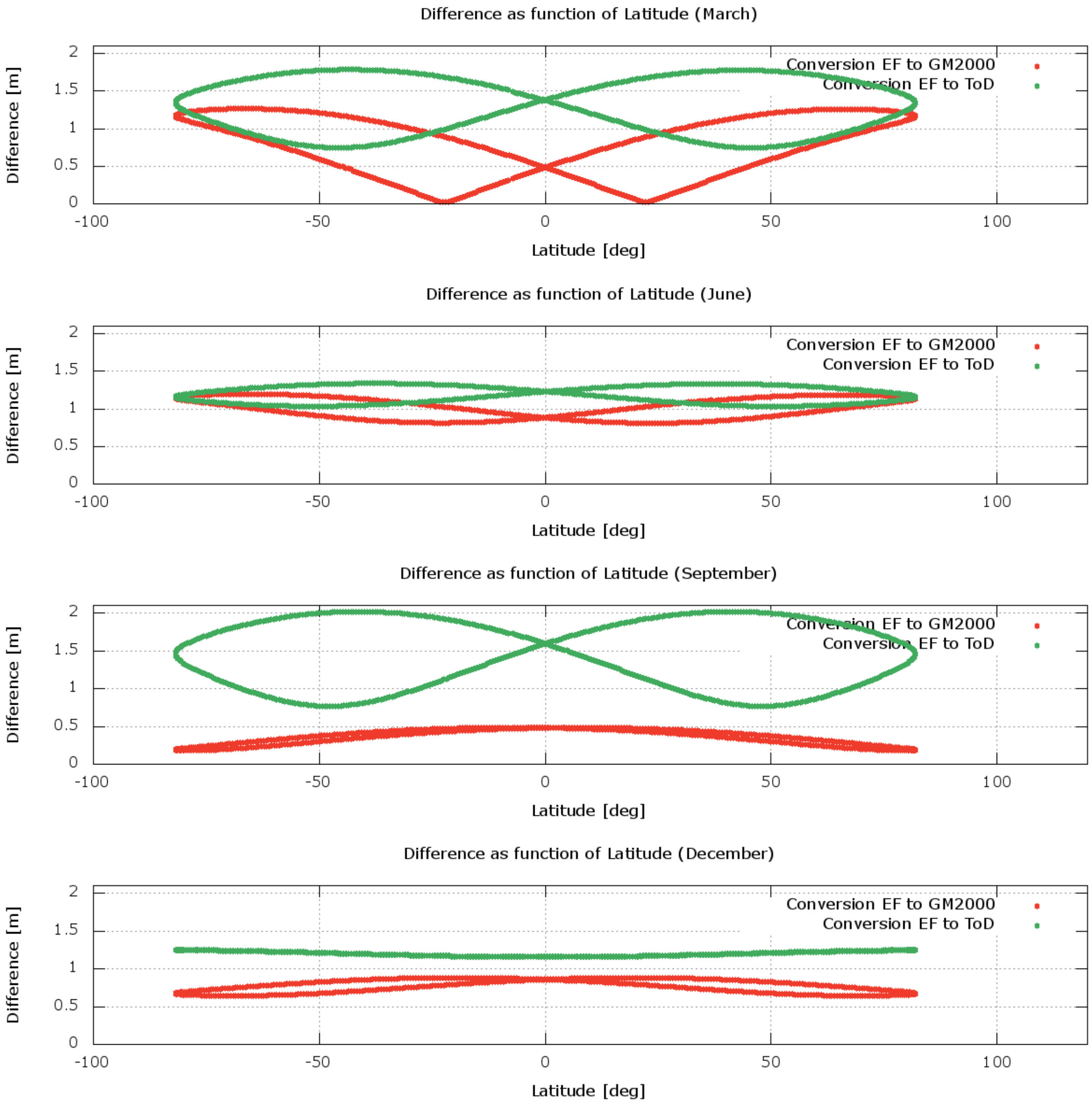


Figure 2 - Difference between vectors converted using EOCFI and SOFA

2.4 Geodesic distance computation

xl_geod_distance computes the distance between two points on the ellipsoid. Its accuracy has been determined by using a third party library, GeographicLib (see [RD11]). The following steps have been executed:

1. given a point *P1 (lon1,lat1)*, a distance *d* and an azimuth *az* (*az = 0* corresponds to the direction to the North), a point *P2 (lon2,lat2)* has been calculated (using GeographicLib) so that *P2* is along direction defined by *P1* and *az* and the geodesic distance between *P1* and *P2* is *d*;
2. the geodesic distance *d_cfi* between *P1* and *P2* has been calculated using **xl_geod_distance**;
3. *d* and *d_cfi* have been compared.

The sequence has been repeated for various latitudes of *P1* (ranging from -80 to 80 deg), distances (1, 10, 100, 100 km) and azimuths (0, 30, 45, 60, 90 deg).

The result of the comparison is given in Figure 3, the difference increases with the distance and is larger with low azimuths, i.e. when the distance is computed along a meridian. The largest difference can be observed with a distance of 1000 km and azimuth 0 deg (North direction) and it is **less than 0.02 meters (worst case).**

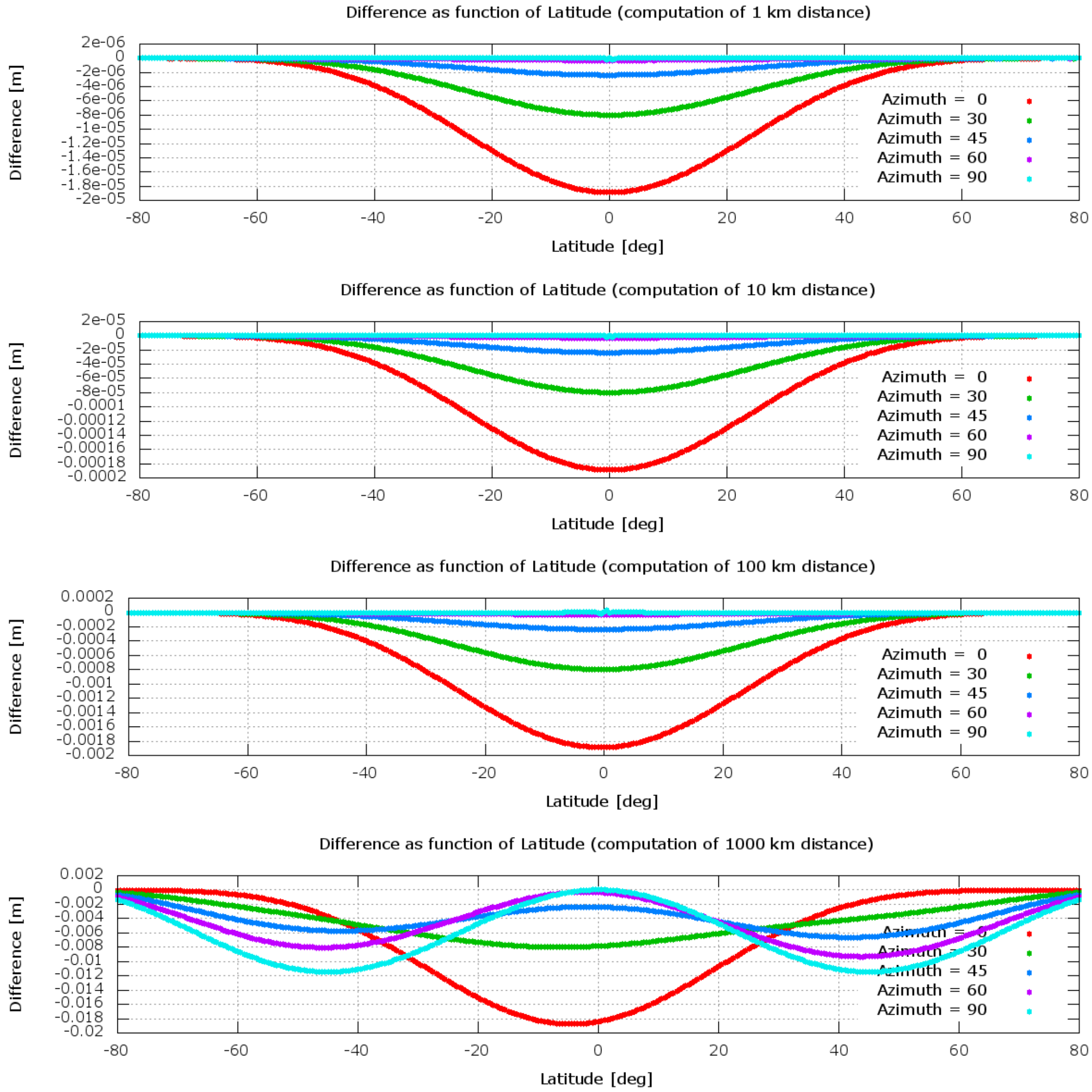


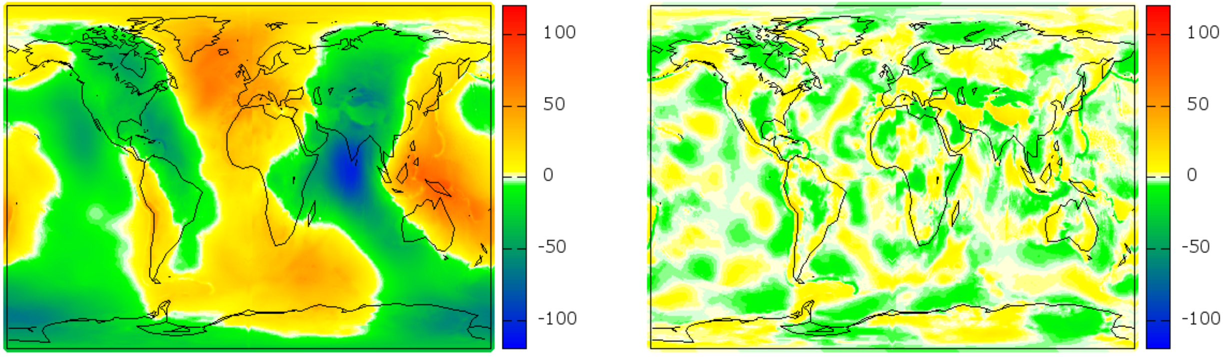
Figure 3 - Difference between computation using EOCFI and GeographicLib

2.5 Geoid computation

xl_geoid_calc computes the geoid at a point of given latitude and longitude. The model used is EGM96, a spherical harmonic model of the Earth's gravitational potential complete to degree and order 360 (see [RD11]). The function receives as input the number of harmonics to be used in the calculation. A number of harmonics smaller than 360 gives a less accurate result and a shorter runtime.

The output of the function has been compared to the exact geoid, as computed using the tools provided in [RD11]. The result is given in Figure 4: plot *A* shows the exact geoid, plots *B, C, D* show the difference between exact geoid and geoid computed using xl_geoid_calc with 10, 30, 60 harmonics respectively.

Table 2 shows the difference between the exact geoid and the output of xl_geoid_calc and the run-time for each type of computation. Using 360 harmonics, the result is identical to the exact geoid at the expense of the computation time. A good compromise between accuracy and run-time can be already reached using 10 or 30 harmonics. The first row of the table indicates the difference in case the geoid is ignored (i.e. is the difference between the exact geoid and a constant value of 0 m).


A - Exact Geoid [m]

B - Exact Geoid vs. Geoid computed with 10 harmonics [m]

C - Exact Geoid vs. Geoid computed with 30 harmonics [m]

D - Exact Geoid vs. Geoid computed with 60 harmonics [m]

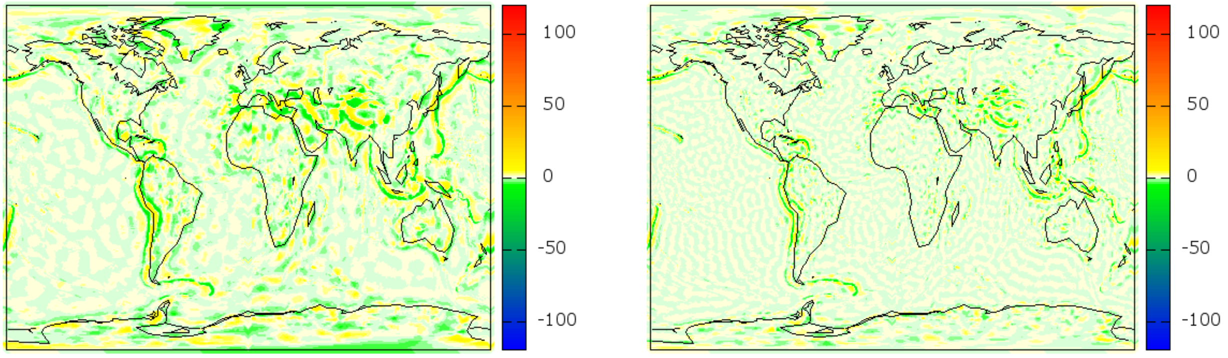


Figure 4 – Geoid computation and comparison with exact geoid

Test Case	Run-time [s] (1000 points)	Difference with Exact Geoid			
		Mean Value [m]	RMS [m]	Min. Value [m]	Max. Value [m]
Geoid not computed	N/A	-1.371	29.286	-106.991	85.391
10 harmonics	0.004	0.024	3.999	-30.226	24.933
30 harmonics	0.030	-0.013	1.970	-21.128	22.597
60 harmonics	0.115	0.001	1.254	-15.726	19.674
180 harmonics	1.042	0.000	0.394	-4.469	6.017
360 harmonics	4.219	0.000	0.000	-0.001	0.001

Table 2 - Geoid computation and comparison with exact geoid

3 ORBIT LIBRARY

3.1 OSV computation

The accuracy of OSV computation (**xo_osv_compute** function) depends on the method used to initialise the orbit_id (orbit propagator or interpolator). The methods that will be analysed are:

- **Orbit Interpolator**: the EOCFI uses this method when the orbit_id is initialised using a list of OSVs (e.g. using function **xo_orbit_init_file** and input one or more reconstituted orbit files). The interpolation algorithm does not require a constant time step between OSVs, however the nominal usage is to use a fixed time step (typically 10s, 30s or 60s). Very small time steps (1 s) are not recommended, due to the limited resolution of the ASCII orbit file format in position (10^{-3} m) and velocity (10^{-6} m/s). Since the interpolated orbit (position and velocity) passes exactly through the provided OSVs, the input OSVs have to be highly accurate. The Orbit interpolation is continuous in position, velocity and acceleration over the complete span of the reconstituted orbit file(s) (an overlap of at least 10 points is required between 2 consecutive files).
- **Keplerian Elements Propagator, Double Mode:** the EOCFI uses this method when the orbit_id is initialised using a list of OSVs at ANX (e.g. when the orbit_id is initialised using function **xo_orbit_init_file** and input a Predicted Orbit File). A weighted average of the propagated OSVs of the previous and next ANX is used to mitigate the effect of the air drag. This orbit propagator is continuous in position, velocity and acceleration over the complete timespan of the orbit file.
- **Keplerian Elements Propagator, Single Mode:** the EOCFI uses this method only when a single OSV at ANX is available for initialisation (e.g. when the orbit_id is initialised using function **xo_orbit_cart_init** or for propagation in the last orbit when using a Predicted Orbit File). This orbit propagator is continuous in position, velocity and acceleration over the propagated period (up to 2 orbits).
- **Keplerian Elements Propagator, Orbit Scenario File (OSF):** the EOCFI uses this method when the orbit_id is initialised with an Orbit Scenario File. The Orbit Scenario File is used to define a simplified reference orbit for planning and simulation purposes.
- **Precise Propagator:** the EOCFI uses this method when the orbit_id is initialized with function **xo_orbit_cart_init_precise** and a single OSV as input. This propagator achieves a better accuracy thanks to spacecraft, space weather and Earth rotation specific input data.

The approach followed to determine the computation accuracy is to calculate the difference between interpolated / propagated OSVs and external data used as reference. Data gathered during the Sentinel-1A commissioning phase from Precise Orbit Determination (POD) service in the period from 18/8/2014 to 1/9/2014 has been used as reference data. OSVs have been computed along one or several orbits and then compared with POD data. The comparison can be expressed in terms of:

- Mean Value, RMS, Maximum value of difference between computed and reference OSVs;
- Mean Value, RMS, Maximum value of along track, across track, radial components of difference between computed and reference OSVs.

The **Mean Value** can be considered an estimation of the **Accuracy** of the method, as it gives an indication of the proximity of the propagated / interpolated value to the reference value.
The **RMS** can be considered an estimation of the **Precision** of the method, as it gives an indication of the dispersion of the propagated / interpolated value to the reference value.

The estimation of accuracy and precision of each method depends on specific characteristics of the mission used as reference (Sentinel-1A in this case). For example:

- The **Orbit Interpolator** can give different results when using POD data (at same time rate) of a satellite orbiting at a lower orbit (e.g. GOCE), as the gravity field produces oscillations with higher amplitude that may not be accurately reconstructed. There are no other dependencies as the algorithm does not use any specific information (i.e. spacecraft, space weather or orbit geometry);
- The **Keplerian Elements Propagator, in Double Mode** has been designed for LEO Sun-Synchronous orbits and tuned for mitigation of the effects of air-drag. Using POD data from non Sun-Synchronous polar orbits during Solar maximum (e.g. Cryosat-2) can give different results;
- The **Keplerian Elements Propagator, Orbit Scenario File (OSF)** assumes rotation symmetry of the orbit, and depends on the duration of the sub-cycle of the orbit. A shorter sub-cycle reduces the building up of Tesseral effects by the Earth Gravity field. Furthermore the OSF assumes that the orbit is maintained by Flight Dynamics. This is not the case for Cryosat-2 where the oscillating inclination can't be corrected by Flight Dynamics, while the OSF assumes a constant inclination.

3.1.1 Orbit Interpolator

The Orbit interpolator requires 10 OSVs (nominally 5 OSVs before and 5 OSVs after the requested time). Therefore it is to be expected that the interpolation degrades when the requested interpolation time is close to the first or last OSV, where the interpolator does not have 5 OSVs on one side.

In order to determine the interpolator accuracy, the following steps have been executed:

1) get a POD file (containing a list of OSVs covering about 2 days at 10s time step);
2) sub-sample this POD file at a time step of 30s (or 60s);
3) using the interpolator and the sub-sampled file as input, compute OSVs at 10s time step;
4) compute the difference between interpolated and reference OSVs (from POD data).

Figure 5 shows the total difference as a function of time in the first hour, for the 30s and the 60s sub-sampling cases. The effect of lack of OSVs can be observed at the very beginning when the interpolation degrades and the difference increases. The red line indicates the numeric resolution of the input file (positions in the POD file are expressed in meters with three significant figures, i.e. millimetres). It can be observed that the 30s case gives a better accuracy. The accuracy improves when the time step decreases. For very small time steps (1 s) the accuracy has not been considered as the numeric resolution might create unwanted noise when using the orbit interpolator with very small steps. Table 3 gives a summary of the results (Degraded OSVs close to first and last OSV have not been taken into account).

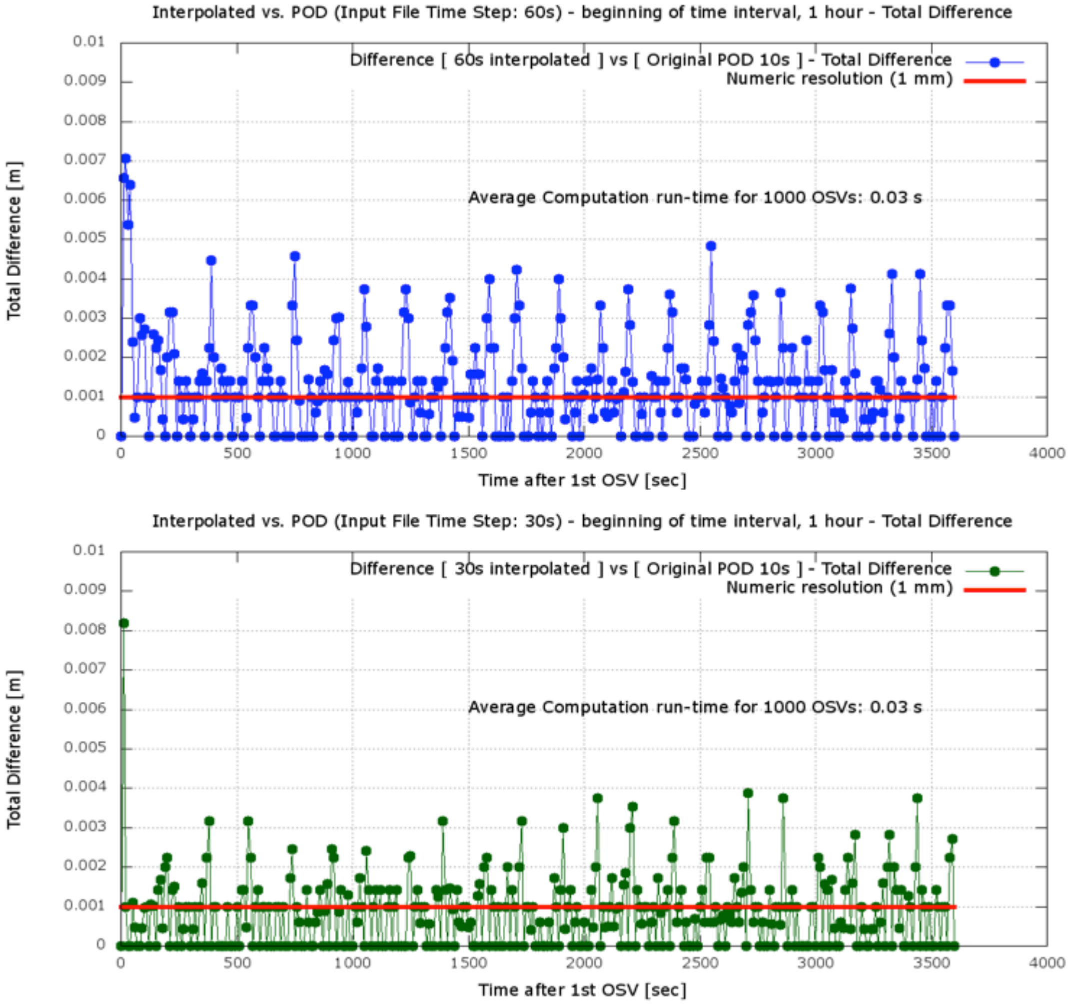


Figure 5 - Interpolator: total difference as function of time

Test Case	Run-time [s] (1000 OSVs)	Orbit Range	Total Difference		
			Mean Value [m]	RMS [m]	Max. Value [m]
30s Time Step	0.03	2161 – 2177	0.0008	0.0009	0.004
60s Time Step	0.03	2161 – 2177	0.0012	0.0012	0.005

Table 3 - Total Difference between interpolated OSVs and reference OSVs

A similar comparison can be done for velocities, see Figure 6 and Table 4. The red line indicates the numeric resolution of the input file (velocities in the POD file are expressed in m/s with six significant figures, i.e. micrometres/s).

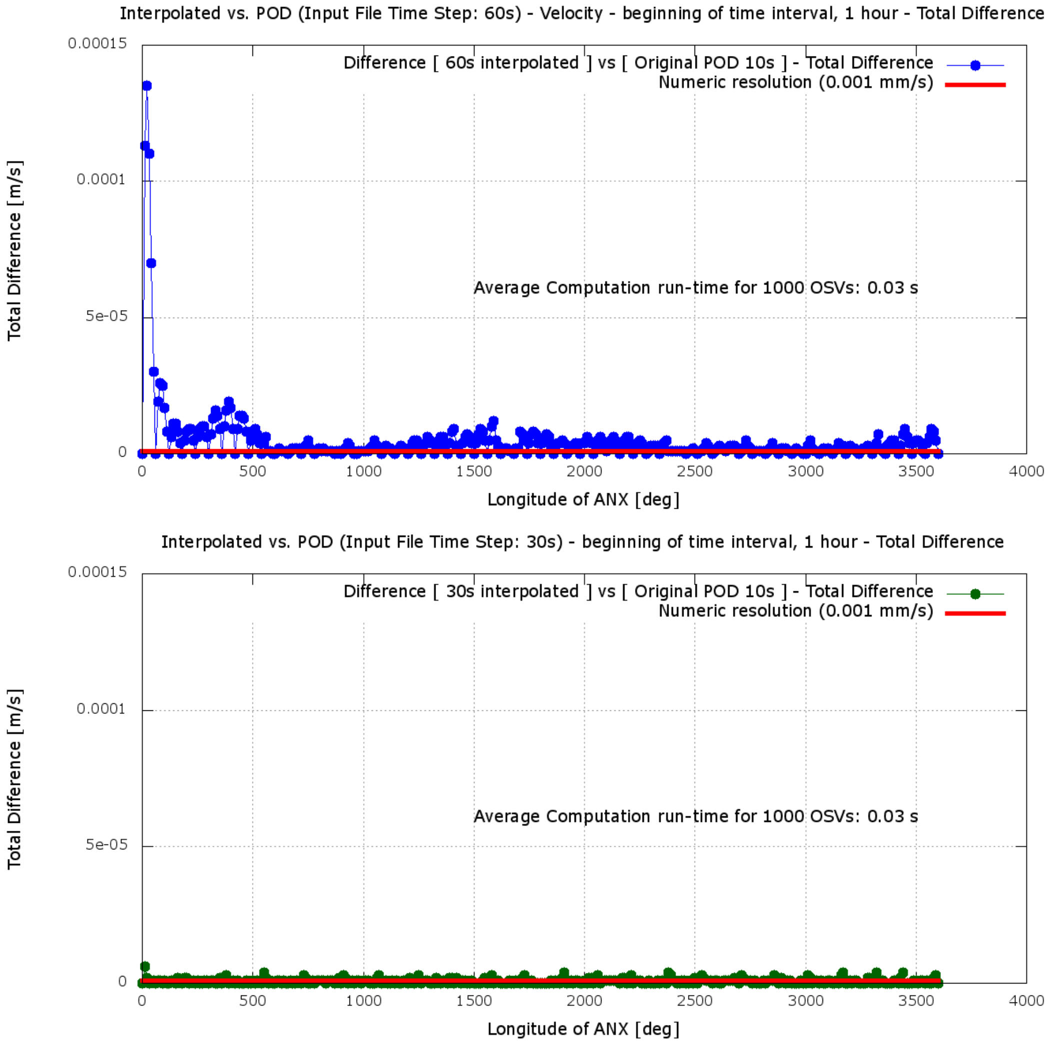


Figure 6 - Interpolator: total difference as function of time (velocity)

Test Case	Run-time [s] (1000 OSVs)	Orbit Range	Total Difference		
			Mean Value [m/s]	RMS [m/s]	Max. Value [m/s]
30s Time Step	0.03	2161 – 2177	0.000001	0.000001	0.000004
60s Time Step	0.03	2161 – 2177	0.000004	0.000004	0.000065

Table 4 - Total Difference between interpolated OSVs and reference OSVs (velocity)

3.1.2 *Keplerian Elements Propagator (double mode)*

In order to determine the accuracy of this propagator, the following steps have been executed:

1. Compute two OSVs at ANX of two consecutive orbits (e.g. orbit N and N+1). Since, as shown in section 3.1, the interpolator accuracy is in the order of the millimetres, such OSVs at ANX have been computed by interpolating the reference POD data exactly at ANX;
2. Generate a Predicted Orbit File with these two OSVs and initialize an orbit_id with this file;
3. Compute OSVs with a time step of 10s along orbit N;
4. Compute the difference between computed and reference OSVs (from POD data);
5. Repeat the steps above for a given range of orbits (covering about 2 weeks).

Figure 7 shows the difference as a function of longitude at ANX in terms of Mean and Max. Value, RMS of total difference (above) and across, along, radial components of difference Mean Value (below). Three discontinuities in the plot can be observed: they are caused by two out-of-plane manoeuvres (65 mm/s each) conducted at consecutive ANXs, resulting in the data for 3 orbits being affected (one small in plane manoeuvre (2 mm/s), conducted close to DNX, cannot be identified in the plots).

Table 5 gives a summary of Mean Value, RMS and Max. value for the total difference (orbits perturbed by manoeuvres have not been taken into account), respectively considering all computed OSVs for a period of about 2 weeks (Overall), only the orbit with the best/worst accuracy (Best/Worst Case).

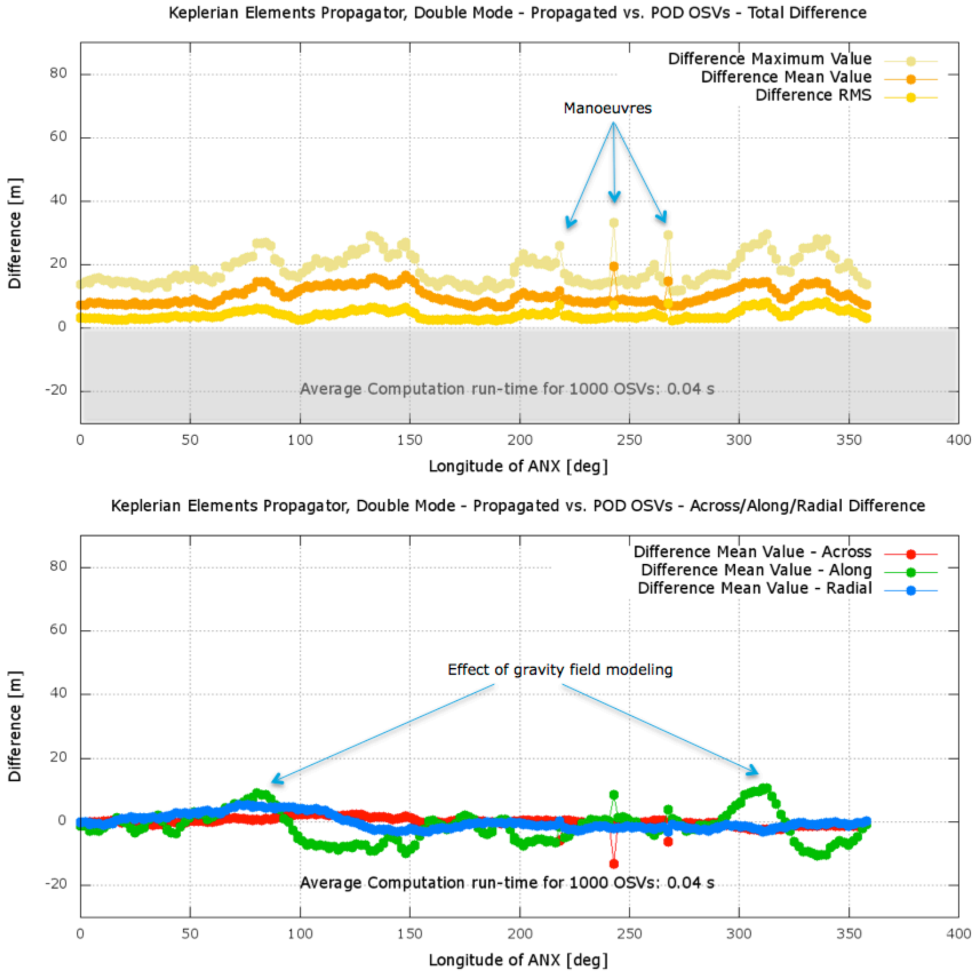


Figure 7 - Double mode propagator: difference as function of ANX longitude

Test Case	Run-time [s] (1000 OSVs)	Orbit Range	Total Difference		
			Mean Value [m]	RMS [m]	Max. Value [m]
Overall	0.04	1987-2205	10.350	5.181	29.656
Best Case	0.04	2107	6.773	2.389	12.633
Worst Case	0.04	2102	14.664	8.972	29.656

Table 5 - Total Difference between propagated OSVs and reference OSVs

The same approach has been used to compare velocities.
Figure 8 shows: the total difference (Mean Value, RMS, Max. value) between velocity vectors in the Earth Fixed Coordinate system (above); the Mean Value of across, along, radial components of relative velocity in inertial frame (below).
Table 6 gives a summary of Mean Value, RMS and Max. value for the total velocity difference (orbits perturbed by manoeuvres have not been taken into account), respectively considering all computed OSVs for a period of about 2 weeks (Overall), only the orbit with the best/worst accuracy (Best/Worst Case).

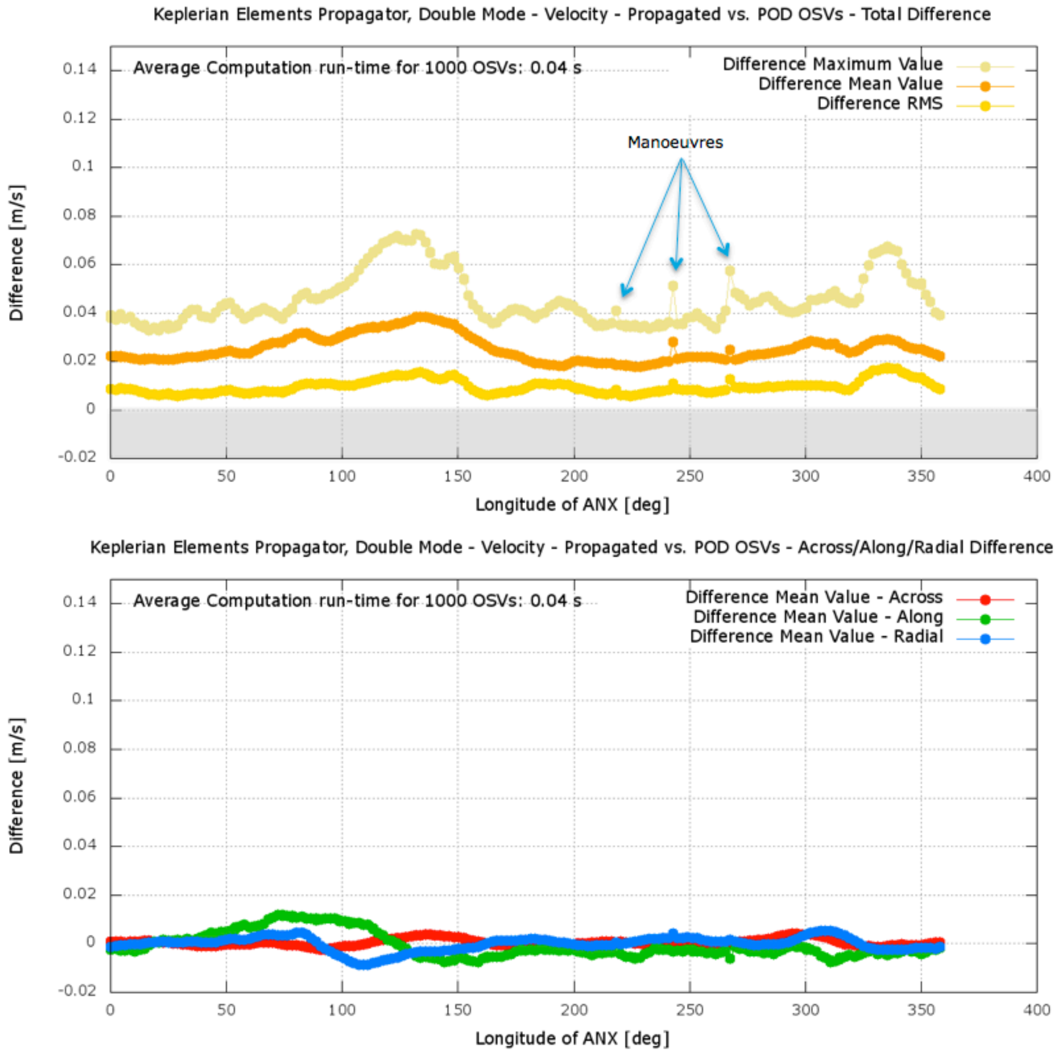


Figure 8 - Double mode propagator: difference as function of ANX longitude (velocity)

Test Case	Run-time [s] (1000 OSVs)	Orbit Range	Total Difference		
			Mean Value [m/s]	RMS [m/s]	Max. Value [m/s]
Overall	0.04	1987-2205	0.025357	0.011387	0.072708
Best Case	0.04	2091	0.018031	0.005832	0.034312
Worst Case	0.04	2153	0.038540	0.015473	0.072124

Table 6 - Total Difference between propagated OSVs and reference OSVs (velocity)

3.1.3 *Keplerian Elements Propagator (single mode)*

In order to determine the accuracy of this propagator, the same sequence of steps described in the previous section has been executed, with the exception of step 1 where only the OSV at ANX of orbit N has been computed.

Again a few discontinuities, corresponding to orbits perturbed by manoeuvres, can be observed (along track component, bottom plot). Two out-of-plane manoeuvres (65 mm/s each) were conducted at consecutive ANXs, resulting in the data for 2 orbits being affected as the OSV's are propagated only forward by this propagator. The small in plane manoeuvre (2 mm/s), conducted close to DNX, cannot be identified in the plot. As the propagation is unconstrained in single mode, and no assumptions have been made or used, especially the along track difference will be bigger compared to the double mode propagation.

Figure 9 shows the difference as a function of longitude at ANX.

Table 7 gives a summary of Mean Value, RMS and Max. value for the total difference (orbits perturbed by manoeuvres have not been taken into account).

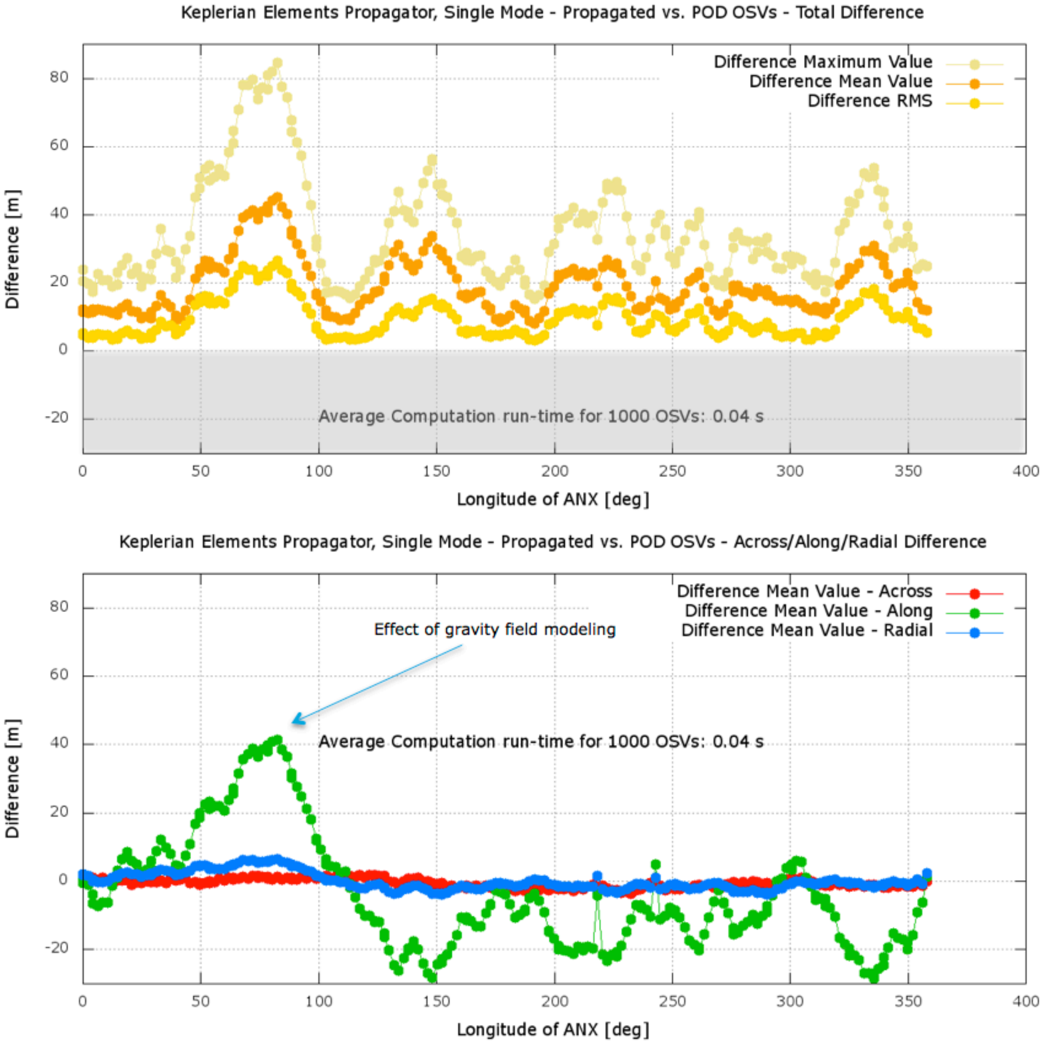


Figure 9 - Single mode propagator: difference as function of ANX longitude

Test Case	Run-time [s] (1000 OSVs)	Orbit Range	Total Difference		
			Mean Value [m]	RMS [m]	Max. Value [m]
Overall	0.04	1987-2205	19.436	13.827	84.481
Best Case	0.04	2034	8.109	3.078	15.345
Worst Case	0.04	2053	45.193	26.378	84.481

Table 7 - Total Difference between propagated OSVs and reference OSVs

The same approach has been used to compare velocities.
Figure 10 shows: the total difference (Mean Value, RMS, Max. value) between velocity vectors in the Earth Fixed Coordinate system (above); the Mean Value of across, along, radial components of relative velocity in inertial frame (below).
Table 8 gives a summary of Mean Value, RMS and Max. value for the total velocity difference (orbits perturbed by manoeuvres have not been taken into account), respectively considering all computed OSVs for a period of about 2 weeks (Overall), only the orbit with the best/worst accuracy (Best/Worst Case).

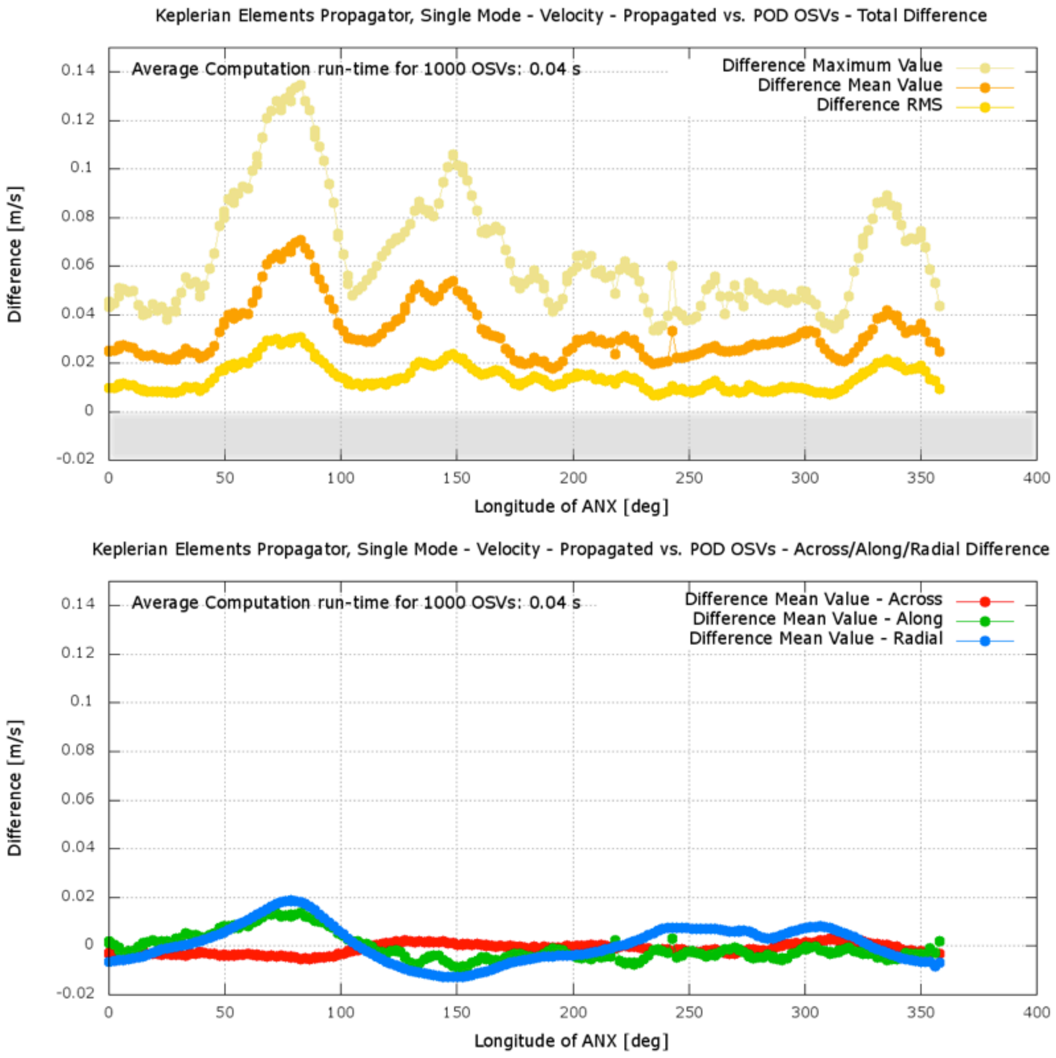


Figure 10 - Single mode propagator: difference as function of ANX longitude (velocity)

Test Case	Run-time [s] (1000 OSVs)	Orbit Range	Total Difference		
			Mean Value [m/s]	RMS [m/s]	Max. Value [m/s]
Overall	0.04	1987-2205	0.032969	0.019514	0.134738
Best Case	0.04	2003	0.020014	0.006859	0.033659
Worst Case	0.04	2053	0.070592	0.030775	0.134738

Table 8 - Total Difference between propagated OSVs and reference OSVs (velocity)

3.1.4 Keplerian Elements Propagator (OSF)

This method is used when the propagator is initialized with an OSF. The OSF defines an orbit according to some parameters that are assumed to be kept constants for all orbits, as for example the nodal period. This is an idealization of the actual situation. In the specific case of POD data for the selected period, an average of 2.5s difference can be observed between UTC times of OSVs at ANX computed respectively using the OSF and using the POD data. This difference is due to the definition of the Mean Local Solar Time (MLST) of 18:00:30, where the actual value (from the POD data) of the MLST has already drifted of about 2.5s.
In order to estimate the accuracy and precision of the propagator alone, this initial bias at ANX has been removed **by applying the same delta UTC time to the propagated OSVs**. However this compensation does not remove the residual difference in longitude and MLST at ANX.

The intended use of the OSF is within the planning process. The plans are made relative to the ANX, either in time or in True Latitude. A difference in MLST will therefore not affect the planning of Earth-fixed targets (instrument acquisitions of the Earth nor ground station passes). The phasing of the OSF is close to the reference orbit as defined by Flight Dynamics, and the usage of the OSF assumes that Flight Dynamics ensures that the actual orbit is close to the reference orbit.

In order to determine the accuracy of this propagator, the following steps have been executed:

1. Initialise the orbit propagator with the current reference OSF;
2. Compute OSVs with a time step of 10s along orbit N;
3. Correct the UTC time of such OSVs as explained above;
4. Compute the difference between computed and reference OSVs (from POD data);
5. Repeat the steps above for a given range of orbits (covering about 2 weeks).

Figure 11 shows the difference as a function of longitude at ANX.
Table 9 gives a summary of Mean Value, RMS and Max. value for the total difference (orbits perturbed by manoeuvres have not been taken into account).

The same approach has been used to compare velocities.

Figure 12 shows: the total difference (Mean Value, RMS, Max. value) between velocity vectors in the Earth Fixed Coordinate system (above); the Mean Value of across, along, radial components of relative velocity in inertial frame (below).

Table 10 gives a summary of Mean Value, RMS and Max. value for the total velocity difference (orbits perturbed by manoeuvres have not been taken into account), respectively considering all computed OSVs for a period of about 2 weeks (Overall), only the orbit with the best/worst accuracy (Best/Worst Case).

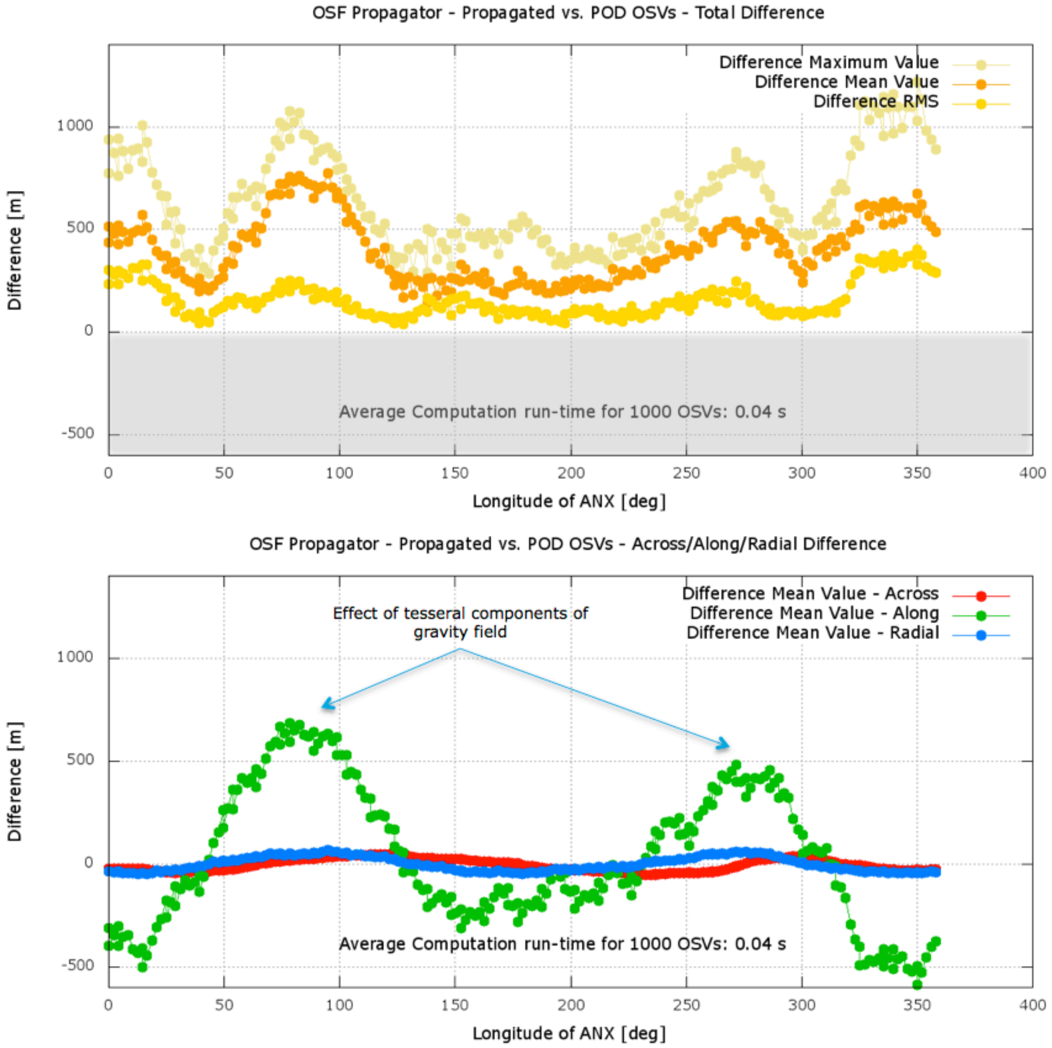


Figure 11 - OSF propagator: difference as function of ANX longitude

Test Case	Run-time [s] (1000 OSVs)	Orbit Range	Total Difference		
			Mean Value [m]	RMS [m]	Max. Value [m]
Overall	0.04	1987-2205	400.811	237.975	1216.617
Best Case	0.04	2022	169.396	39.647	233.585
Worst Case	0.04	2188	672.262	403.620	1216.617

Table 9 - Total Difference between propagated OSVs and reference OSVs

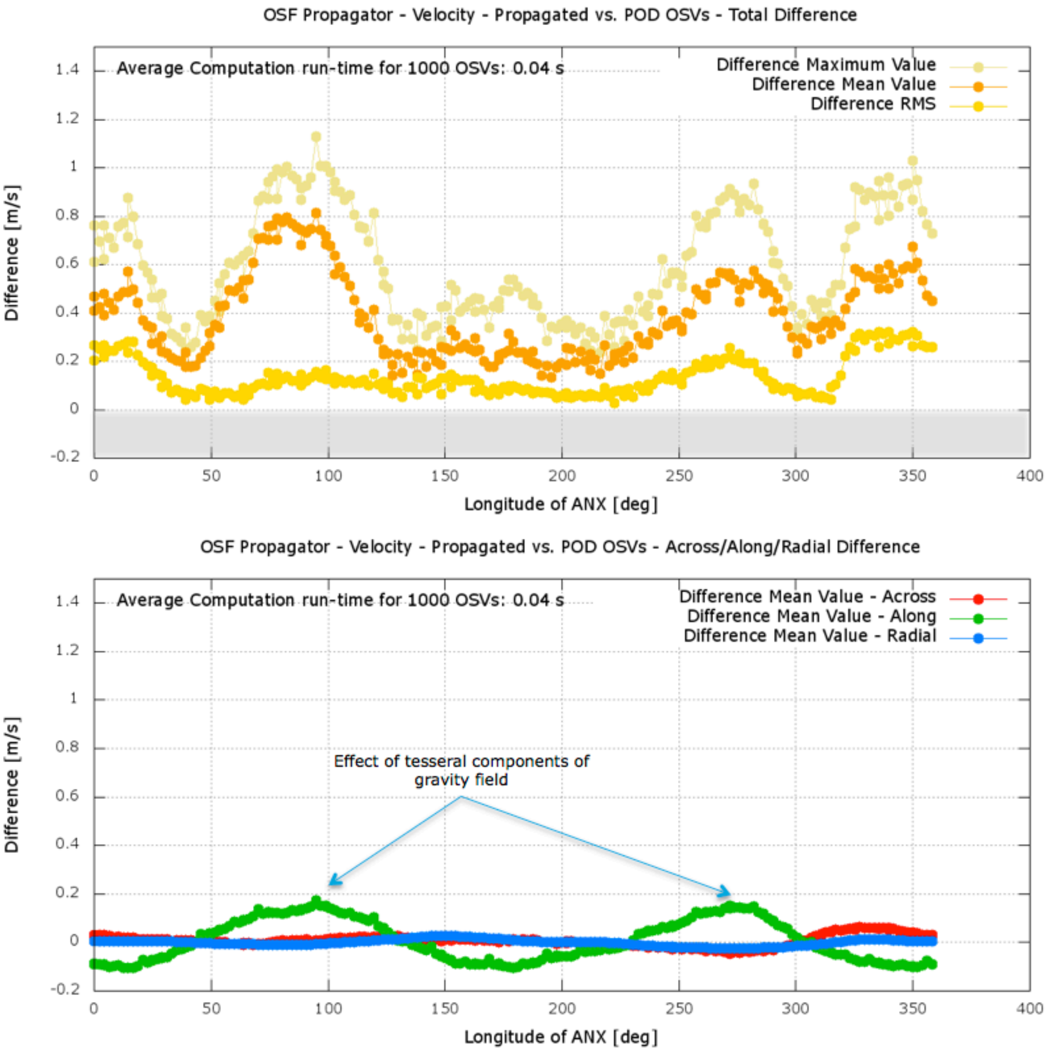


Figure 12 - OSF propagator: difference as function of ANX longitude (velocity)

Test Case	**Run-time [s] (1000 OSVs)**	**Orbit Range**	**Total Difference**		
			Mean Value [m/s]	**RMS [m/s]**	**Max. Value [m/s]**
Overall	0.04	1987-2205	0.388723	0.230822	1.128739
Best Case	0.04	1989	0.205023	0.026343	0.25801
Worst Case	0.04	2188	0.672065	0.320147	1.029442

Table 10 - Total Difference between propagated OSVs and reference OSVs (velocity)

3.1.5 *Precise Propagator*

The precise propagator has to be initialized with a single OSV at any position along the orbit. In order to compare with other propagators, the same OSVs at ANX computed for the single mode propagator are used also for this propagator.

The precise propagator parameters have been specifically tuned according to the current mission (Sentinel-1A), current solar activity and other. The time_id (the Software object holding data necessary for co-ordinate systems transformations) has been initialised with the IERS Bulletin B (see [RD08]) that provides the most accurate estimation of the polar motion.

In order to determine the accuracy of this propagator, the following steps have been executed:

1. Compute the OSVs at ANX of a given orbit N (by interpolating the reference POD data exactly at ANX) and initialise the precise propagator with such OSVs;
2. Compute 12 OSVs equally time-spaced along orbit N (due to long run-time of this propagator, a reduced number of OSVs equally spaced along the orbit are calculated);
3. Compute the difference between computed and reference OSVs (from POD data);
4. Repeat the steps above for a given range of orbits (covering about 2 weeks).

Figure 13 shows the difference as a function of longitude at ANX. Again a few discontinuities, corresponding to orbits perturbed by manoeuvres, can be observed. Two out-of-plane manoeuvres (65 mm/s each) were conducted at consecutive ANXs, resulting in the data for 2 orbits being affected as the OSV's are propagated only forward by this propagator. Also the small in plane manoeuvre (2 mm/s), conducted close to DNX, can be identified in the plot. Table 11 gives a summary of Mean Value, RMS and Max. value for the total difference (orbits perturbed by manoeuvres have not been taken into account).

Compared to the propagators presented in the previous sections, the precise propagator has a better accuracy at the expense of a longer run-time.

The same approach has been used to compare velocities.

Figure 14 shows: the total difference (Mean Value, RMS, Max. value) between velocity vectors in the Earth Fixed Coordinate system (above); the Mean Value of across, along, radial components of relative velocity in inertial frame (below).

Table 12 gives a summary of Mean Value, RMS and Max. value for the total velocity difference (orbits perturbed by manoeuvres have not been taken into account), respectively considering all computed OSVs for a period of about 2 weeks (Overall), only the orbit with the best/worst accuracy (Best/Worst Case).

Figure 15 and Table 13 show the result of the same step sequence (only for position), but without using the IERS bulletin. It can be observed that the difference has increased due to the inaccuracy of co-ordinate systems transformations (information about the current polar motion is not taken into account).

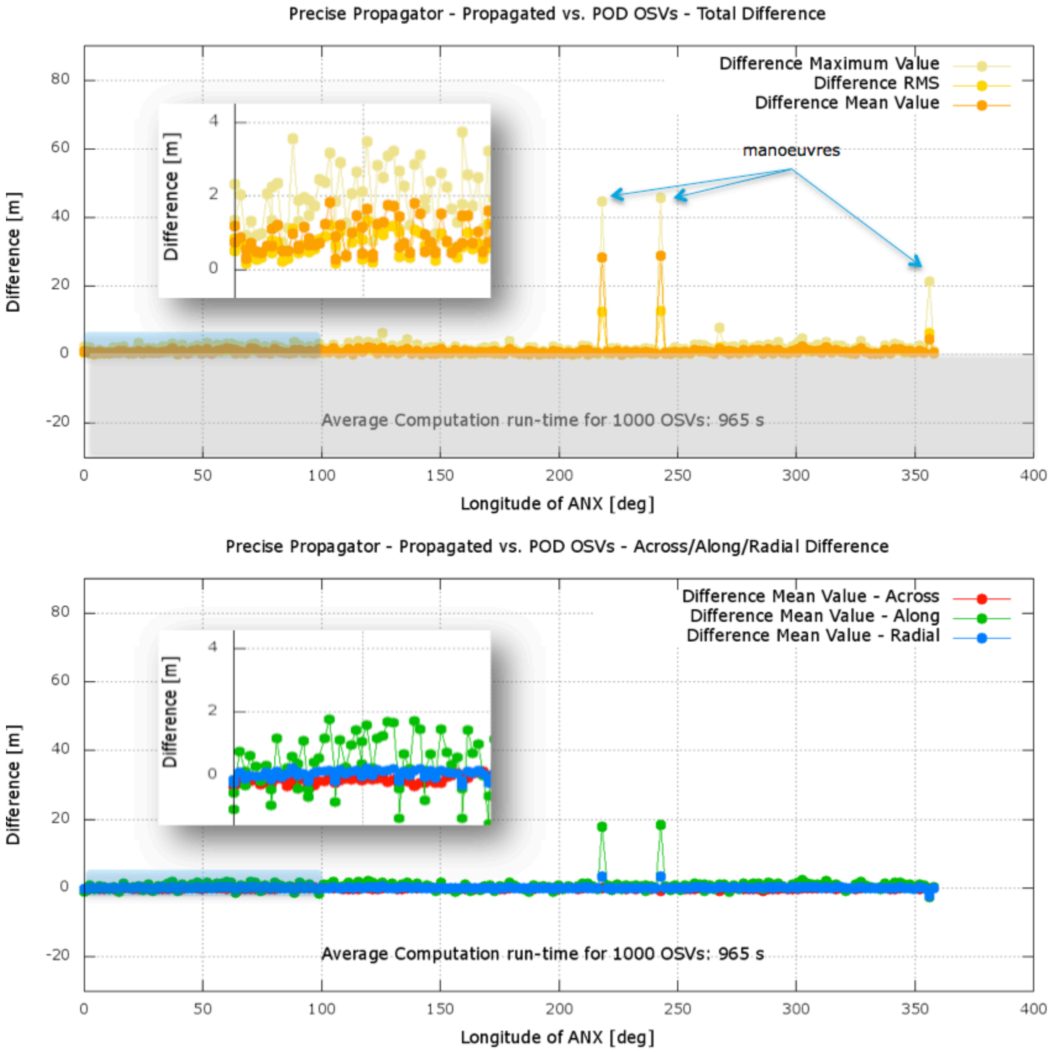


Figure 13 - Precise propagator: difference as function of ANX longitude

Test Case	Run-time [s] (1000 OSVs)	Orbit Range	Total Difference		
			Mean Value [m]	RMS [m]	Max. Value [m]
Overall	965	1987-2205	0.892	0.871	6.270
Best Case	965	2018	0.177	0.106	0.370
Worst Case	965	2095	1.778	2.117	6.270

Table 11 - Total Difference between propagated OSVs and reference OSVs

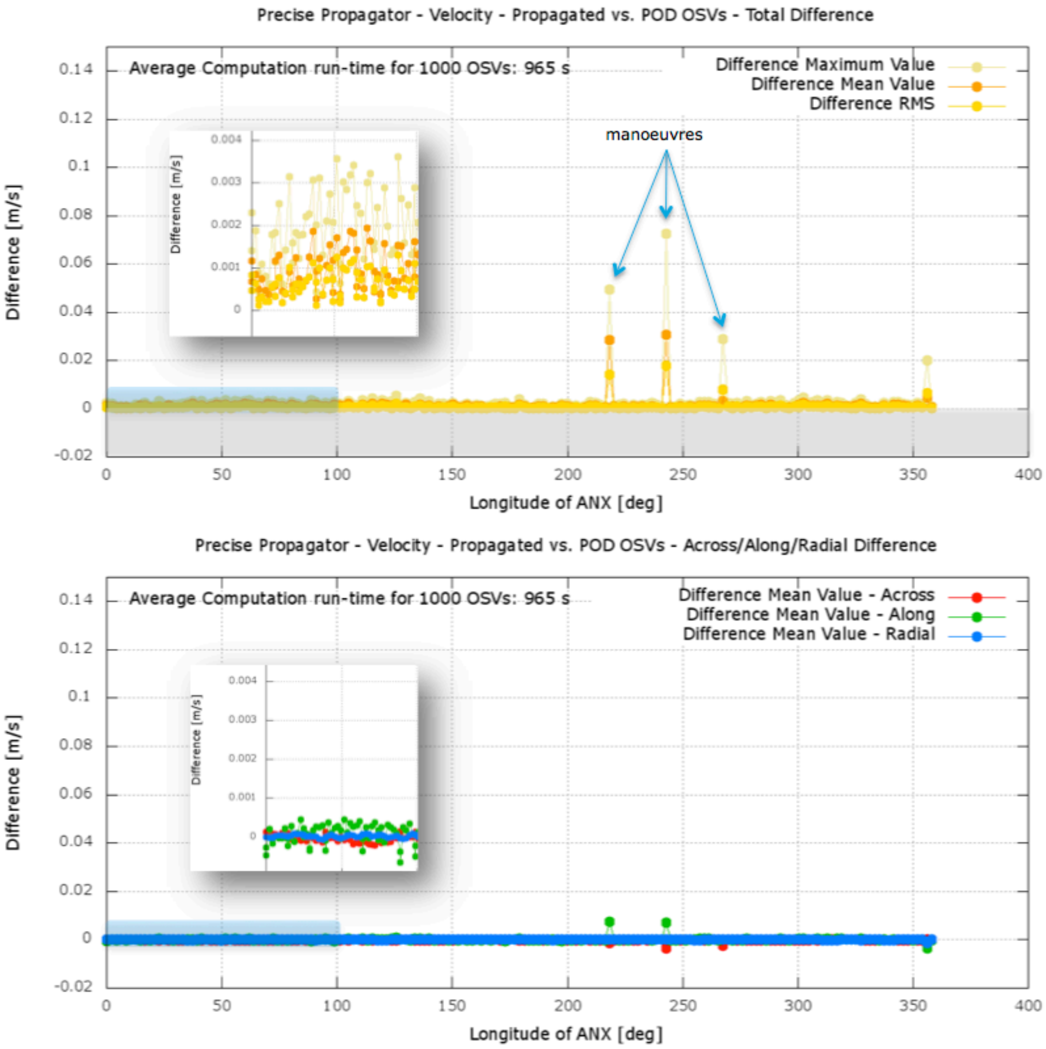


Figure 14 - Precise propagator: difference as function of ANX longitude (velocity)

Test Case	Run-time [s] (1000 OSVs)	Orbit Range	Total Difference		
			Mean Value [m/s]	RMS [m/s]	Max. Value [m/s]
Overall	965	1987-2205	0.000883	0.000828	0.005474
Best Case	965	2183	0.000184	0.000101	0.000321
Worst Case	965	2095	0.001779	0.001925	0.005474

Table 12 - Total Difference between propagated OSVs and reference OSVs (velocity)

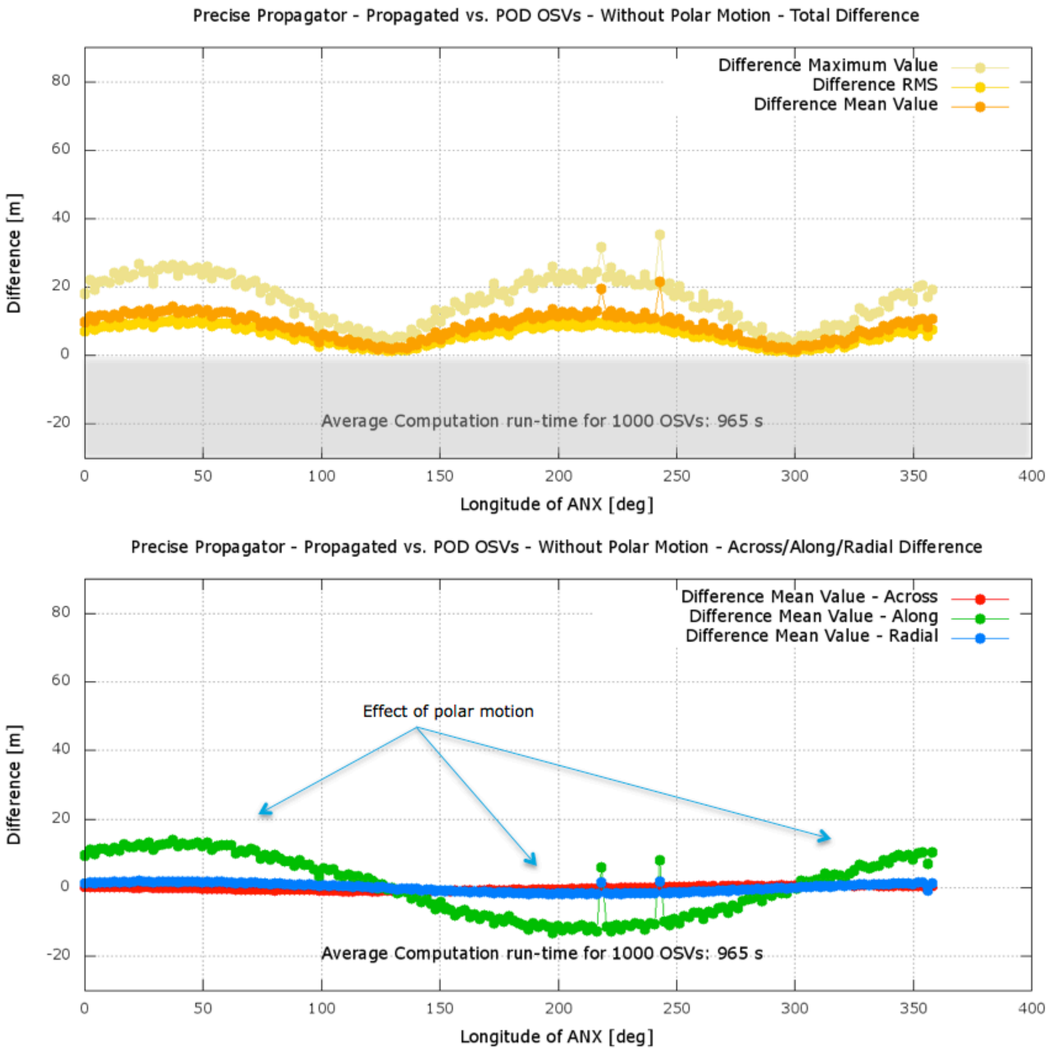


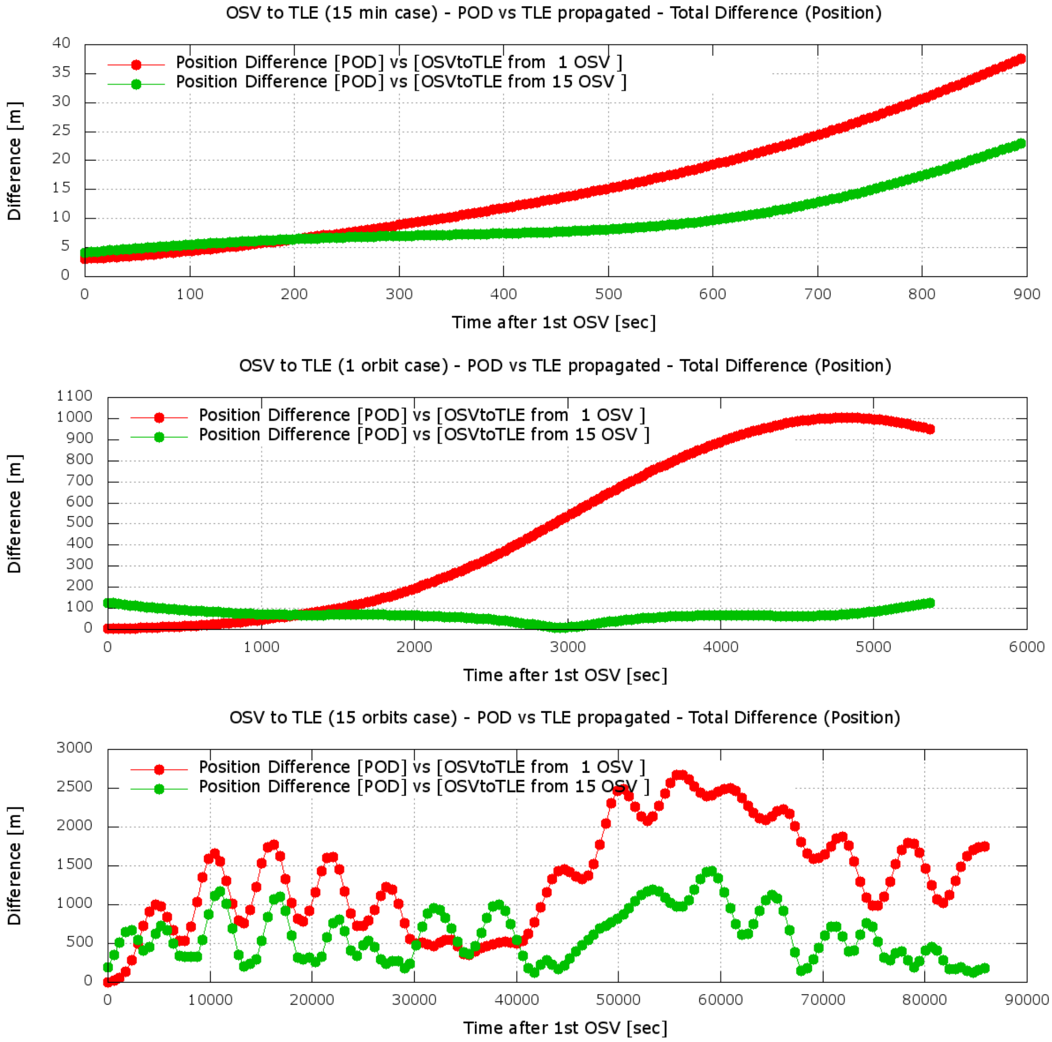
Figure 15 - Precise propagator: difference as function of ANX longitude (without polar motion)

Test Case	Run-time [s] (1000 OSVs)	Orbit Range	Total Difference		
			Mean Value [m]	RMS [m]	Max. Value [m]
Overall	965	1987-2205	8.347	7.576	26.861
Best Case	965	2190	1.538	1.110	3.608
Worst Case	965	2084	14.221	10.422	26.604

Table 13 - Total Difference between propagated OSVs and reference OSVs (without polar motion)

3.2 Conversion from OSV to TLE

A Two Line Element (TLE) is a file containing the parameters required to run the SGP4 propagator (see [RD08]). The **xo_osv_to_tle** function receives as input a list of OSVs and computes the best fitting TLE. The accuracy of this function can be determined as follows: a list of OSVs is extracted from a Sentinel-1A POD file and then passed to **xo_osv_to_tle** for the TLE computation; the output TLE is used to initialise an orbit_id and compute by propagation a new set of 150 OSVs (with function xo_osv_compute that implements the SGP4 propagator in case of TLE initialisation); the computed OSVs are compared to those obtained by interpolating the POD file at the corresponding times. Three different scenarios are considered: a short period of 15 minutes; one full orbit; 15 orbits (approximately 1 day).


OSV to TLE (15 min case) - POD vs TLE propagated - Total Difference (Position)

OSV to TLE (1 orbit case) - POD vs TLE propagated - Total Difference (Position)

OSV to TLE (15 orbits case) - POD vs TLE propagated - Total Difference (Position)

Figure 16 – OSV to TLE accuracy (position):
15 minutes (top), 1 orbit (middle), bottom (15 orbits)

Test Scenario	Test Case	Total Difference		
		Mean Value [m]	RMS [m]	Max. Value [m]
15 minutes	1 OSV	15.553	9.858	37.554
	15 OSVs	9.686	4.715	22.967
1 orbit	1 OSV	471.180	383.069	1003.055
	15 OSVs	67.127	25.107	127.312
15 orbits	1 OSV	1352.966	682.052	2675.433
	15 OSVs	601.272	327.988	1431.584

Table 14 – OSV to TLE accuracy summary (position)

For each scenario, the step sequence has been repeated using a different number of input OSVs: only one OSV (at the beginning of the time interval); 15 OSVs equally spaced in time along the time interval.

Figure 16 shows the difference between propagated and reference OSVs (position) as function of time for the three scenarios. The red curve is the evolution of the difference when only one OSV is used: the difference is small at the beginning (since the TLE has been computed based on the first OSV) but then it increases due to the simplified model used by the SGP4 propagator. The green curve is related to the 15 OSVs case: thanks to the additional OSVs, the difference remains within smaller limits along the whole time interval.

Table 15 gives a summary of Mean Value, RMS and Max. value of the difference computed with the various scenarios and number of input OSVs.

A similar comparison can be made for the velocity (see Figure 17 and Table 15).

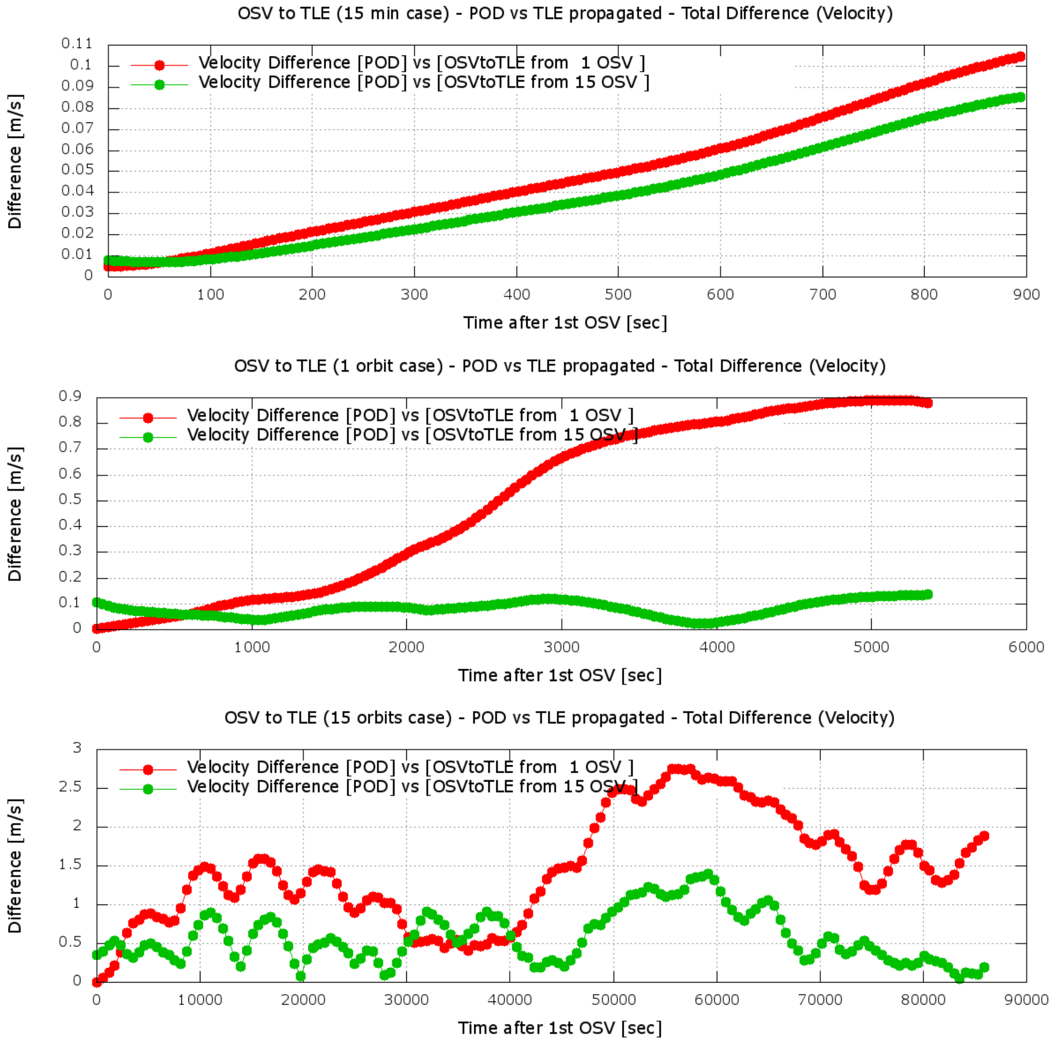


Figure 17 – OSV to TLE accuracy (velocity):
15 minutes (top), 1 orbit (middle), bottom (15 orbits)

Test Scenario	Test Case	Total Difference		
		Mean Value [m/s]	RMS [m/s]	Max. Value [m/s]
15 minutes	1 OSV	0.048135	0.029250	0.104565
	15 OSVs	0.038305	0.024244	0.085502
1 orbit	1 OSV	0.489935	0.327090	0.891143
	15 OSVs	0.080392	0.029642	0.135446
15 orbits	1 OSV	1.446500	0.694590	2.752718
	15 OSVs	0.579683	0.328487	1.394294

Table 15 – OSV to TLE accuracy summary (velocity)

3.3 Conversion of position on orbit (OPS angle) to UTC time

xo_position_on_orbit_to_time computes the UTC time at which a given position on orbit (e.g. OPS angle) is reached along a given orbit. Its accuracy has been determined as follows:

1) compute an OSV (by propagation) at UTC *T1*;
2) compute (with function xl_position_on_orbit) the position on orbit *OPS* correspondent to such OSV;
3) compute UTC *T2* using **xo_position_on_orbit_to_time** with *OPS* as input position on orbit;
4) compare *T1* and *T2*.

The steps above have been repeated for several OSV along one orbit. Figure 18 shows the difference between *T1* and *T2* as function of position on orbit. The difference is always **below 18 microseconds**.

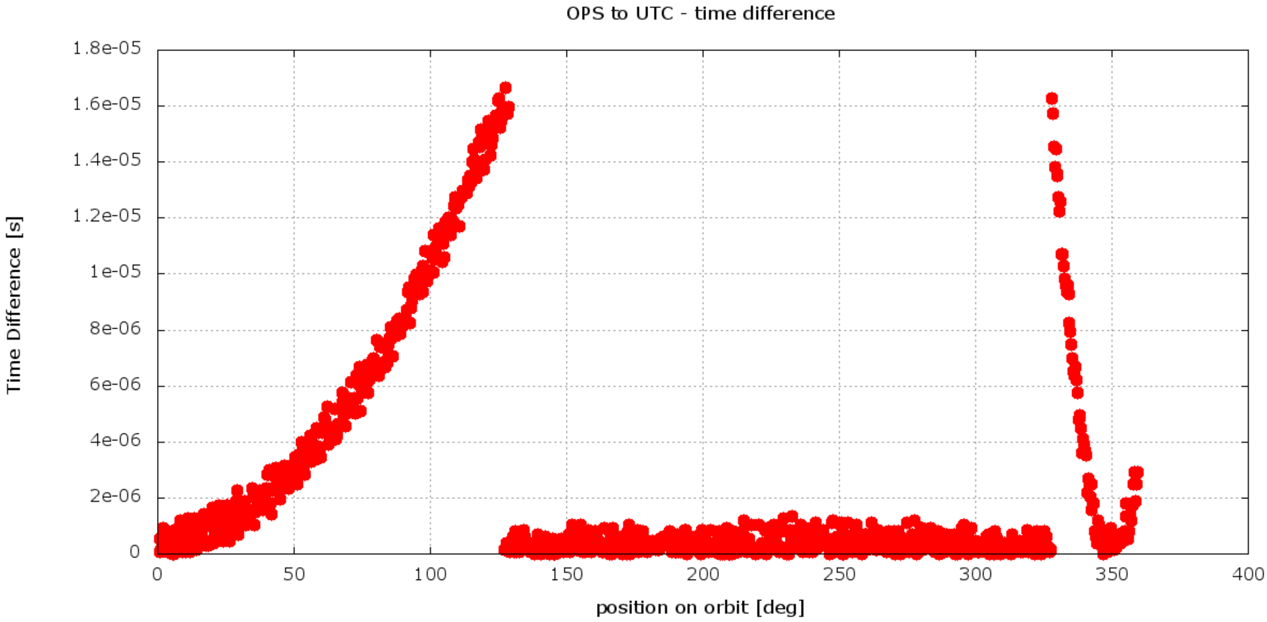


Figure 18 – OPS to UTC, time difference as function of OPS

ANNEX A: LIST OF LIB AND ORBIT FUNCTIONS

LIB library Function(s)	Description	Outputs	Algorithm Type	Accuracy Determination	Document Reference
xl_time_ref_init_file xl_time_ref_init xl_time_id_init xl_time_close xl_time_get_id_data xl_time_set_id_data xl_run_init xl_run_get_ids xl_run_close xl_time_get_leap_second_info	functions for time_id initialisation and manipulation	time_id and/or its internal data	N/A	N/A	N/A
xl_time_ascii_to_ascii xl_time_ascii_to_processing xl_time_ascii_to_transport xl_time_processing_to_ascii xl_time_processing_to_processing xl_time_processing_to_transport xl_time_transport_to_ascii xl_time_transport_to_processing xl_time_transport_to_transport	time conversion	time in different format and/or reference	Analytical	Not Required	N/A
xl_default_sat_init xl_default_sat_close	sat_Id manipulation	sat_id	N/A	N/A	N/A
xl_set_tle_sat_data	TLE propagator configuration	None	N/A	N/A	N/A
xl_model_init xl_model_close xl_model_get_data	model_id manipulation	model_id and its internal data	N/A	N/A	N/A
xl_time_add xl_time_diff	time manipulation	time sum/ difference in processing (MJD2000) format	Analytical	Not Required	N/A
xl_time_obt_to_time xl_time_time_to_obt	time conversion from/to Orbit Time to UTC	UTC in processing format / Orbit Time	Analytical	Not Required	N/A
xl_change_cart_cs	Change Coordinate System of a Vector	pos vel acc	Model	Required	PE-TN-ESA-GS-404 section 2.3
xl_geod_to_cart	Conversion from Geodetic to Cartesian	pos vel	Analytical	Not Required	N/A
xl_cart_to_geod	Conversion from Cartesian to Geodetic	lon lat h	Iterative	Required	PE-TN-ESA-GS-404 section 2.2
xl_kepl_to_cart	Conversion from Keplerian Elements to OSV	pos vel	Analytical	N/A	N/A
xl_cart_to_kepl	Conversion from OSV to Keplerian Elements	Semi major Axis Eccentricity Inclination Right Ascension of Ascending Node Argument of Perigee	Analytical	N/A	N/A

LIB library Function(s)	Description	Outputs	Algorithm Type	Accuracy Determination	Document Reference
		Mean Anomaly			
xl_cart_to_radec	Transformation From equatorial cartesian coordinates to right ascension and declination. or From galactic cartesian coordinates to galactic longitude and latitude.	Right ascension (or galactic longitude) Declination (or galactic latitude) Proper motion in the right ascension Proper motion in the declination Radial velocity Parallax	Analytical	Not Required	N/A
xl_radec_to_cart	From right ascension and declination to equatorial cartesian coordinates. Or From galactic longitude and latitude to galactic cartesian coordinates.	pos vel	Analytical	Not Required	N/A
xl_topocentric_to_ef	transforms topocentric azimuth and elevation to the Earth Fixed Reference frame.	Cartesian Direction Cartesian Change in Direction	Analytical	Not Required	N/A
xl_ef_to_topocentric	transforms Earth Fixed coordinates to topocentric coordinates for a given ground position.	Azimuth Elevation Range Azimuth Rate Elevation Rate Range Rate	Analytical	Not Required	N/A
xl_sun xl_moon xl_planet	calculates the position and velocity vector of the Sun / Moon / Planets in the Earth Fixed coordinate system.	pos vel	Model	Required	PE-TN-ESA-GS-404 section 2.1
xl_star_radec	calculates the right ascension and declination of a star in the True of Date coordinate system.	ra dec	Analytical	Not Required	N/A
xl_star_catalog	calculates the right ascension and declination of a star in a selected star catalogue.	ra dec	Analytical	Not Required	N/A

LIB library Function(s)	Description	Outputs	Algorithm Type	Accuracy Determination	Document Reference
xl_geod_distance	calculates the geodesic distance between two points that lay on the same ellipsoid, and the azimuth of the related geodesic line at both points	distance azimuth from point 1 to 2 and 2 to 1	Iterative	Required	PE-TN-ESA-GS-404 section 2.4
xl_euler_to_matrix	conversion from euler angles to rotation matrix	rotation matrix	Analytical	Not Required	N/A
xl_matrix_to_euler	conversion from rotation matrix to euler angles	euler angles	Analytical	Not Required	N/A
xl_position_on_orbit	angle describing the position of the satellite within the orbit (OPS angle)	angle angle rate angle rate rate	Analytical	N/A	N/A
xl_get_rotation_angles	calculates the rotation angles between two sets of orthonormal right-handed unit vectors expressed wrt an identical coordinate frame	rotation angles rool-pitch-yaw	Analytical	Not Required	N/A
xl_get_rotated_vectors	calculates the rotated unit vectors given a set of unit vectors and the rotation angles expressed wrt an identical coordinate frame	rotated vectors X,Y,Z	Analytical	Not Required	N/A
xl_quaternions_to_vectors	calculates the orthonormal unit vectors from a given set of quaternions	vectors X,Y,Z	Analytical	Not Required	N/A
xl_vectors_to_quaternions	calculates the set of quaternions that correspond to a set of orthonormal unit vectors	quaternions q1,q2,q3,q4	Analytical	Not Required	N/A
xl_geoid_calc	computes the geoid undulation, that is, the height of the geoid over the ellipsoid	geoid undulation	Model	Required	PE-TN-ESA-GS-404 section 2.5

Table 16 – List of functions in the LIB library

ORBIT library Function(s)	Description	Outputs	Algorithm Type	Accuracy Determination	Document Reference
xo_orbit_init_def xo_orbit_init_def_2 xo_orbit_cart_init xo_orbit_cart_init_precise xo_orbit_id_init xo_orbit_init_file xo_orbit_init_file_precise xo_orbit_init_geo xo_orbit_close xo_orbit_init_status xo_orbit_get_sat_id xo_orbit_get_mode xo_orbit_get_osv xo_orbit_set_osv xo_orbit_get_anx xo_orbit_set_anx xo_orbit_get_osf_rec xo_orbit_set_osf_rec xo_orbit_get_val_time xo_orbit_set_val_time xo_orbit_get_osv_compute_validity xo_orbit_get_propag_mode xo_orbit_get_propag_config xo_orbit_get_interpol_mode xo_orbit_get_interpol_config xo_orbit_id_clone xo_orbit_get_precise_propag_config xo_orbit_set_precise_propag_config xo_orbit_get_geo_orbit_info xo_orbit_set_geo_orbit_info xo_orbit_id_init_data_close xo_run_init xo_orbit_id_change	functions for orbit_id initialisation and manipulation	orbit_id and/or its internal data	N/A	N/A	N/A
xo_osv_compute (Interpolator)	OSV computation	pos	CFI specific	Required	PE-TN-ESA-GS-404 section 3.1.1
		vel	CFI specific	Required	PE-TN-ESA-GS-404 section 3.1.1
		acc	Analytical	Not Required	N/A
xo_osv_compute (Keplerian Elements Propagator, Double mode)	OSV computation	pos	CFI specific	Required	PE-TN-ESA-GS-404 section 3.1.2
		vel	CFI specific	Required	PE-TN-ESA-GS-404 section 3.1.2
		acc	Analytical	Not Required	N/A
xo_osv_compute (Keplerian Elements Propagator, Single mode)	OSV computation	pos	CFI specific	Required	PE-TN-ESA-GS-404 section 3.1.3
		vel	CFI specific	Required	PE-TN-ESA-GS-404 section 3.1.3
		acc	Analytical	Not Required	N/A

ORBIT library Function(s)	Description	Outputs	Algorithm Type	Accuracy Determination	Document Reference
xo_osv_compute (Keplerian Elements Propagator, OSF)	OSV computation	pos	CFI specific	Required	PE-TN-ESA-GS-404 section 3.1.4
		vel	CFI specific	Required	PE-TN-ESA-GS-404 section 3.1.4
		acc	Analytical	Not Required	N/A
xo_osv_compute (precise propagator)	OSV computation	pos	CFI specific	Required	PE-TN-ESA-GS-404 section 3.1.5
		vel	CFI specific	Required	PE-TN-ESA-GS-404 section 3.1.5
		acc	Analytical	Not Required	N/A
xo_osv_compute (TLE propagator)	OSV computation	pos	Analytical	Not Required	N/A
		vel	Analytical	Not Required	N/A
		acc	Analytical	Not Required	N/A
xo_osv_compute_run xo_osv_compute_extra_run xo_orbit_to_time_run xo_time_to_orbit_run xo_orbit_info_run xo_orbit_abs_from_rel_run xo_orbit_abs_from_phase_run xo_orbit_rel_from_abs_run xo_run_init xo_gen_osf_create_run xo_gen_osf_create_run_2 xo_gen_osf_append_orbit_change_run xo_gen_osf_append_orbit_change_run_2 xo_gen_osf_change_repeat_cycle_run xo_gen_osf_change_repeat_cycle_run_2 xo_gen_osf_add_drift_cycle_run xo_gen_pof_run xo_gen_rof_run xo_gen_rof_prototype_run xo_gen_dnf_run xo_check_osf_run xo_check_oef_run	“run“ functions	See “non-run” function	See “non-run“ function	See “non-run“ function	See “non-run“ function
xo_osv_compute_extra	OSV computation extra results	• Nodal period • ANX Time • Absolute Orbit Number • Time since ANX (sec) • Mean Kepler Elements of computed OSV • lon, lat, h (geodetic co-ordinates) of satellite • lon, lat, h (geodetic co-ordinates) rates of satellite • lon, lat, h (geodetic co-ordinates) rates-rates of satellite	Analytical Some outputs are model based (e.g. True Local Solar time) or iterative (e.g. geodetic coordinates). Their accuracy is	Not required	N/A

ORBIT library Function(s)	Description	Outputs	Algorithm Type	Accuracy Determination	Document Reference
		• Radius of curvature parallel to meridian at the SSP (EF frame) • Radius of curvature orthogonal to meridian at the SSP (EF frame) • Radius of curvature along groundtrack at the SSP (EF frame) • Northward component of the velocity relative to the Earth of the SSP (Topocentric frame) • Eastward component of the velocity relative to the Earth of the SSP (Topocentric frame) • Magnitude of the velocity relative to the Earth of the SSP (Topocentric frame) • Azimuth of the velocity relative to the Earth of the SSP (Topocentric frame) • Northward component of the accelera tion relative to the Earth of the SSP (Topocentric frame) • Eastward component of the acceleration relative to the Earth of the SSP (Topocentric frame) • Groundtrack tangential component of the acceleration relative to the Earth of the SSP (Topocentric frame) • Azimuth of the acceleration relative to the Earth of the SSP (Topocentric frame) • Satellite eclipse flag • Sun Zenith Angle • MLST at SSP • True local solar time at the SSP • True Sun's (centre) right ascension (TOD frame) • True Sun's (centre) declination (TOD frame) • True Sun's semi-diameter • Moon's (centre) right ascension (TOD frame) • Moon's (centre) declination (TOD frame) • Moon's semi-diameter • Area of Moon lit by Sun • Osculating Keplerian elements of the OSV (TOD frame) • Orbit radius (TOD frame) • Radial orbit velocity component (TOD frame)	identical to the one of functions in LIB library such as xl_sun and xl_cart_to_geod.		

ORBIT library Function(s)	Description	Outputs	Algorithm Type	Accuracy Determination	Document Reference
		• "Transversal orbit velocity component (TOD frame) • Right ascension of the satellite (TOD frame) • Declination of the satellite (TOD frame) • Earth rotation angle [H] • Right ascension rate of the satellite (TOD frame) • Right ascension rate-rate of the satellite (TOD frame) • Satellite osculating true latitude (EF frame) • Satellite osculating true latitude rate (EF frame) • Satellite osculating true latitude rate-rate (EF frame)			
xo_orbit_to_time xo_time_to_orbit	Conversion from UTC to orbit time and vice-versa	UTC time / orbit time	Analytical	Not Required	N/A
xo_orbit_info	Computation of Orbit information	Repeat Cycle Cycle Length MLST Drift MLST ANX Longitude ANX UTC position at ANX velocity at ANX mean Keplerian elements at ANX osculating Keplerian elements at ANX	Analytical	Not Required	N/A
xo_osv_to_tle	Conversion from OSV to TLE	TLE	CFI specific	Required	PE-TN-ESA-GS-404 section 3.2
xo_orbit_abs_from_rel xo_orbit_abs_from_phase xo_orbit_rel_from_abs	Conversion between absolute and relative orbits (using OSF)	orbit number (absolute or relative)	Analytical	Not Required	N/A
xo_gen_osf_create xo_gen_osf_create_2 xo_gen_osf_create_run_2 xo_gen_osf_append_orbit_change xo_gen_osf_append_orbit_change_2 xo_gen_osf_change_repeat_cycle xo_gen_osf_change_repeat_cycle_2 xo_gen_osf_change_repeat_cycle_run_2 xo_gen_osf_add_drift_cycle	Functions for creating and modifying an OSF	OSF	Analytical	Not Required	N/A
xo_gen_pof xo_gen_rof xo_gen_rof_prototype xo_gen_dnf xo_gen_tle xo_check_osf xo_check_oef	Functions for creating and checking various types of files	ROF / POF / DNF / TLE	N/A	N/A	N/A

ORBIT library Function(s)	Description	Outputs	Algorithm Type	Accuracy Determination	Document Reference
xo_position_on_orbit_to_time	Convert position on orbit (OPS angle) to UTC time	UTC time	Iterative	Required	PE-TN-ESA-GS-404 section 3.3
xo_orbit_data_filter	orbit data filter (e.g. outlier detection).	orbit data (e.g. OSVs)	CFI specific	N/A	N/A

Table 17 – List of functions in the ORBIT library