Earth Observation Mission CFI Software

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

1 SCOPE

This document describes in detail the time references and formats, reference frames, parameters, models, and units that will be used by the Earth Observation Mission CFI Software. The description sometimes goes beyond the CFI-needed information, when deemed necessary for the sake of a correct explanation.

All topics treated along the document are applicable to the following CFI libraries:

- EO_DATA_HANDLING
- EO_LIB
- EO_ORBIT
- EO_POINTING
- EO_VISIBILITY

The present document covers the different LEO satellite missions considered in the frame of the ESA Earth Observation Programme.

2 ACRONYMS AND NOMENCLATURE

2.1 Acronyms

ADM	Atmospheric Dynamics Mission
ANX	Ascending Node Crossing
AOCS	Attitude and Orbit Control Sub-system
CFI	Customer Furnished Item
DORIS	Doppler Orbitography and Radio positioning Integrated by Satellite
ESA	European Space Agency
ESTEC	European Space Technology and Research Centre
EO	Earth Observation
EOCFI	Earth Observation CFI Software
FK4	Fourth Fundamental Catalogue
FK5	Fifth Fundamental Catalogue
GOCE	Gravity Field and Steady-state Ocean Circulation Mission
GPS	Global Positioning System
IAU	International Astronomical Union
IERS	International Earth Rotation Service
IRF	Instrument Reference Frame
IRM	IERS Reference Meridian
IRP	IERS Reference Pole
ITRF	IERS Terrestrial Reference Frame
JD	Julian Day
LOS	Line of Sight
LNP	Local Normal Pointing
MLST	Mean Local Solar Time
MJD2000	Modified Julian Day of 2000
OBT	On-Board Time

SBT	Satellite Binary Time
SGP4	Simplified General Perturbations Satellite Orbit Model 4
SNRF	Satellite Nominal Reference Frame
SOF	Satellite Orbital Frame
SRF	Satellite Reference Frame
S/C	Spacecraft
SI	International System of Units
SSP	Sub-Satellite Point
TAI	International Atomic Time
TLE	Two Line Elements
TLST	True Local Solar Time
UT1	Universal Time UT1
UTC	Coordinated Universal Time
YSM	Yaw Steering Mode

3 APPLICABLE AND REFERENCE DOCUMENTS

3.1 Applicable Documents

3.2 Reference Documents

ALMAN95	The Astronomical Almanac for the year 1995.
ALMAN05	The Astronomical Almanac for the year 2005.
BOWRING	Method of Bowring. NGT Geodesia 93-7. P 333-335. 1993.
CELES	www.celestrak.com
FLANDERN	Low-precision formulae for planetary positions. Astrophysical Journal Supplement Series: 41. P 391-411. T.C.Van Flandern, K.F. Pulkkinen. November 1979.
GREEN	Spherical Astronomy. Green, R.M. 1985
HEISKANEN	Physical Geodesy. Weikko A. Heiskanen, Helmut Moritz. Graz 1987.
IERS_SUPL	Explanatory Supplement to IERS Bulletins A and B. International Earth Rotation Service (IERS). March 1995.
KLINKRAD	Semi-Analytical Theory for Precise Single Orbit Predictions of ERS-1. ER-RP-ESA-SY-004. H.K. Klinkrad (ESA/ESTEC/WMM). Issue 1.0. 28/06/87.
LIU_ALFORD	Semianalytic Theory for a Close-Earth Artificial Satellite. Journals of Guidance and Control Vol. 3, No 4. J.J.F. Liu and R.L. Alford. July-August 1980.
MCD_SP	Earth Observation Mission CFI Software MISSION SPECIFIC CUSTOMIZATIONS. EO-MA-DMS-GS-018. Issue 1.0 27/10/2009
OAD_TIME	OAD Standards: Time and Coordinate Systems for ESOC Flight Dynamics Operations. Orbit Attitude Division, ESOC. Issue 1. May 1994.
STD76	U.S. Standard Atmosphere 1976. National Oceanic and Atmosphere Administration.
EDLEN	“Handbook of geophysics and the space environment”. Adolph S. Jursa. Air Force Geophysics Laboratory, 1985.

4 TIME REFERENCES AND MODELS

4.1 Time References

The time references which may be used in the context of the Earth Observation missions are listed in table:

Table 1: Earth Observation time reference definitions

Time reference	Usage Examples
Universal Time (UT1)	Typically used as time reference for orbit state vectors.
Universal Time Coordinated (UTC)	Typically used as time reference for all products datation.
International Atomic Time (TAI)	DORIS products.
GPS Time	Typically used by missions having a GPS receiver on-board (eg. GOCE, ADM, Sentinel 1, Sentinel 2, Sentinel 3).

The relationships between UT1, UTC and TAI are illustrated in the following figure:

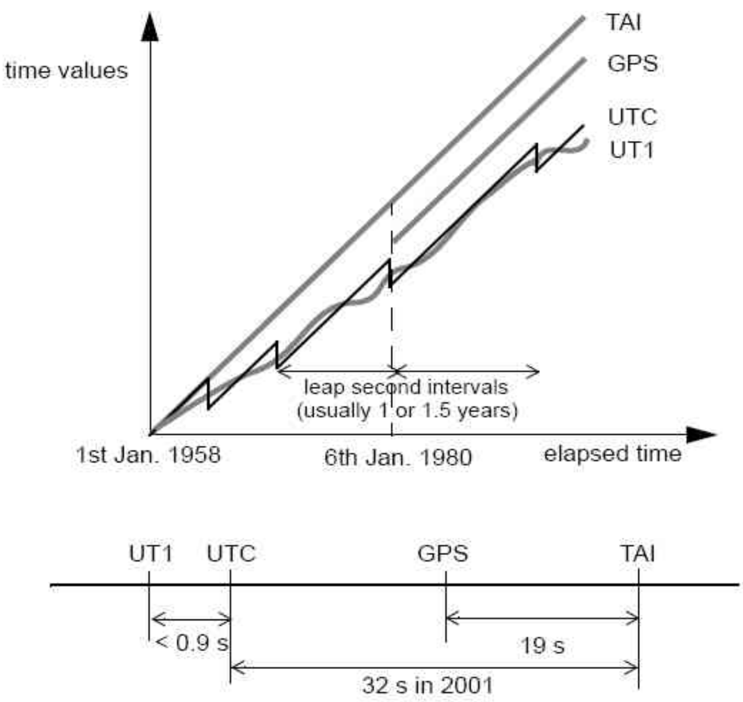


Figure 1: Relationships between UT1, UTC and TAI

Universal Time **(UT1)** is a time reference that conforms, within a close approximation, to the mean diurnal motion of the Earth. It is determined from observations of the diurnal motions of the stars, and then corrected for the shift in the longitude of the observing stations caused by the polar motion.

The time system generally used is the ***Coordinated Universal Time*** **(UTC)**, previously called *Greenwich Mean Time*. The UTC is piece wise uniform and continuous, i.e. the time difference between UTC and TAI is equal to an integer number of seconds and is constant except for occasional jumps from inserted integer *leap seconds*. The leap seconds are inserted to cause UTC to follow the rotation of the Earth, which is expressed by means of the non uniform time reference *Universal Time* UT1.

If UT1 is predicted to lag behind UTC by more than 0.9 seconds, a leap second is inserted. The message is distributed in a *Special Bulletin C* by the International Earth Rotation Service (IERS).

The insertion of leap seconds is scheduled to occur with first preference at July 1st and January 1st at 00:00:00 UTC, and with second preference at April 1st and October 1st at 00:00:00 UTC.

ΔUT1 = UT1 - UTC is the increment to be applied to UTC to give UT1, expressed with a precision of 0.1 seconds, and which is broadcasted, and any change announced in a Bulletin D, by the IERS[1].

DUT1 is the predicted value of ΔUT1. Predictions of UT1 - UTC daily up to ninety days, and at monthly intervals up to a year in advance, are included in a Bulletin A which is published weekly by the IERS.

International Atomic Time **(TAI)** represents the mean of readings of several atomic clocks, and its fundamental unit is exactly one SI second at mean sea level and is, therefore, constant and continuous.

ΔTAI = TAI - UTC is the increment to be applied to UTC to give TAI.

GPS Time is an atomic clock time similar to but not the same as UTC time. It is synchronised to UTC but the main difference relies in the fact that GPS time does not introduce any leap second. Thus, the introduction of UTC leap second causes the GPS time and UTC time to differ by a known integer number of cumulative leap seconds; i.e. the leap seconds that have been accumulated since GPS epoch in midnight January 5, 1980.

ΔGPS = TAI - GPS is the increment to be applied to GPS to give TAI, being a constant value of 19 seconds.

4.2 Time formats

The ***Julian Day*** **(JD)** is the interval of time in days and fraction of a day since 4713 BC January 1 at Greenwich noon (12:00:00).

The ***Modified Julian Day 2000*** **(MJD2000)** is the interval of time in days and fraction of day since 2000 January 1 at 00:00:00.

JD = MJD2000 + 2451544.5 [decimal days]

The time format year, month, day of month, hour, minute and second follows the Gregorian calendar.

4.2.1 Earth Observation time formats

The time formats used with the time references proposed in section 4.1 can be one of the following:

- Processing
- Transport
- ASCII

1 *ΔUT1 usually changes 1-2 ms per day*

Table 2: Earth Observation time formats

Time format		Description	Usage
Processing		64-bits floating point number, for decimal days	Internal processing, such as product processing sequences. Only for continuous times, i.e. TAI
Transport	EO CFI Standard	Three 32-bits integer numbers for days, seconds and microseconds[2]	Time values exchange between computers
ASCII[3]	Standard	Text string: “yyyy-mm-dd_hh:mm:ss“	Readable input/output, such as file headers, log messages, ...
	Standard with reference	Text string: “RRR=yyyy-mm-dd_hh:mm:ss“	
	Standard with microseconds	Text string: “yyyy-mm-dd_hh:mm:ss.uuuuuu“	
	Standard with reference and microseconds	Text string: “RRR=yyyy-mm-dd_hh:mm:ss.uuuuuu“	
	Compact	Text string: “yyyymmdd_hhmmss“	
	Compact with reference	Text string: “RRR=yyyymmdd_hhmmss“	
	Compact with microseconds	Text string: “yyyymmdd_hhmmssuuuuuu“	
	Compact with reference and microseconds	Text string: “RRR=yyyymmdd_ hhmmssuuuuuu“	
	CCSDS-A	Text string: “yyyy-mm-ddThh:mm:ss“	
	CCSDS-A with reference	Text string: “RRR=yyyy-mm-ddThh:mm:ss“	
	CCSDS-A with microseconds	Text string: “yyyy-mm-ddThh:mm:ss.uuuuuu“	
	CCSDS-A with reference and microseconds	Text string: “RRR=yyyy-mm-ddThh:mm:ss.uuuuuu“	

2 This is the EO CFI Standard Transport Format. Additional Transport Formats have been defined for specific missions, see [MCD_SP].

3 These are the EO CFI Standard ASCII Formats. Additional ASCII Formats have been defined for specific missions, see [MCD_SP].

4.3 Time resolution

The time resolution is one microsecond.

4.4 Earth Observation On-board times

The **On Board Time** is the time maintained by the Spacecraft and is the time reference for all spacecraft on-board activities. Depending upon the purpose and requirements of the mission, the time format used on-board the satellite will be different. See [MCD_SP] for mission specific on-board time definitions.

5 REFERENCE FRAMES

The following reference frames are used in the context of Earth Observation missions:

Table 3: Earth Observation reference frames usage

Reference frame	Usage Examples
Galactic	Star position and velocities can be given in this reference frame
Barycentric Mean of 1950	Some star catalogues use this reference frame to express the positions of their stars.
Barycentric Mean of 2000	The star catalogues usually use this reference frame to express the positions of their stars.
Heliocentric Mean of 2000	The ephemeris of the planets are usually expressed in this reference frame.
Geocentric Mean of 2000	The FOCC performs the internal calculations related to the predicted and restituted orbits in this reference frame.
Mean of Date	The Mean Local Solar Time is defined in this reference frame.
True of Date	It is the inertial reference frame used for input and output in the CFI software (e.g. star positions).
Earth fixed	It is the reference frame used for input and output of the satellite state vector (i.e. orbit definition), and for the output for geolocation in the CFI software.
Pseudo Earth Fixed	It is similar to Earth Fixed but without considering polar motion rotation.
Topocentric	It is the local horizontal reference frame used to define a looking direction.
Satellite Orbital	It is a reference frame centred in the satellite and defined by the satellite position and velocity. Its used as a reference for the application of the selected attitude control mode.
Satellite Nominal Attitude	It is used for the attitude determination. It is based on relation with the Satellite Orbital frame.
Satellite Attitude	It is used for the attitude determination as well. It is based on relation the Satellite Nominal Attitude frame or on measurements.
Instrument Attitude	It is the reference frame that constitutes the reference for the definition of a look direction relative to the satellite (e.g. to express the pointing of an instrument).

5.1 General Reference Frames

5.1.1 Galactic

The galactic plane is determined by the statistical study of the galactic dynamics. In this reference frame, position are determined by a galactic latitude and longitude. The galactic latitude are taken as the angle measured from the galactic plane, while the galactic longitude are measured from the direction of the galactic centre.

In order to relate the galactic coordinates of a star to its equatorial coordinates, it is necessary to know the position of the galactic pole and the position of the galactic centre. These points have been adopted as follow, for the epoch 1950.0:

Right ascension of the Galactic pole = 12h 49m.

Declination of the Galactic pole = 27º.4.

Galactic longitude of the north celestial pole = 123º (also known as the position angle of the galactic centre)

5.1.2 Barycentric Mean of 1950

It is based on the star catalogue FK4 for the epoch B1950, since the directions of its axes are defined relatively to a given number of that star catalogue positions and proper motions.

The centre of this reference frame is the barycentre of the Solar System. The x-y plane coincides with the predicted mean Earth equatorial plane at the epoch B1950, and the x-axis points towards the predicted mean vernal equinox. The latter is the intersection of the mean equator plane with the mean ecliptic, and the ecliptic is the orbit of the Earth around the Sun. The z-axis points towards north.

The word *mean* indicates that the relatively short periodic nutations of the Earth are smoothed out in the calculation of the mean equator and equinox.

5.1.3 Barycentric Mean of 2000

It is based, according to the recommendations of the International Astronomical Union (IAU), on the star catalogue FK5 for the epoch J2000.0, since the directions of its axes are defined relatively to a given number of that star catalogue positions and proper motions.

The accuracy of this reference system, realized through the FK5 star catalogue, is approximately 0.1’’.

The centre of this reference frame is the barycentre of the Solar System. The x-y plane coincides with the predicted mean Earth equatorial plane at the epoch J2000.0, and the x-axis points towards the predicted mean vernal equinox. The latter is the intersection of the mean equator plane with the mean ecliptic, and the ecliptic is the orbit of the Earth around the Sun. The z-axis points towards north.

The word *mean* indicates that the relatively short periodic nutations of the Earth are smoothed out in the calculation of the mean equator and equinox.

5.1.4 Heliocentric Mean of 2000

It is obtained by a parallel translation of the Barycentric Mean of 2000.0 reference frame from the barycenter of the Solar System to the centre of the Sun.

5.1.5 Geocentric Mean of 2000

It is obtained by a parallel translation of the Barycentric Mean of 2000.0 reference frame from the barycenter of the Solar System to the centre of the Earth.

5.1.6 Mean of Date

The centre of this reference frame is the centre of the Earth. The x-y plane and the x-axis are defined by the mean Earth equatorial plane and the mean vernal equinox of date. The expression *mean of date* means that the system of coordinate axes are rotated with the Earth´s precession from J2000.0 to the date used as epoch. The z-axis points towards north.

The precession of the Earth is the secular effect of the gravitational attraction from the Sun and the Moon on the equatorial bulge of the Earth.

5.1.7 True of Date

The centre of this reference frame is the centre of the Earth. The x-y plane and the x-axis are defined by the true Earth equatorial plane and the true vernal equinox of date. The expression *true of date* indicates the instantaneous Earth equatorial plane and vernal equinox. The transformation from the Mean of Date to the True of Date is the adopted model of the nutation of the Earth.

The nutation is the short periodic effect of the gravitational attraction of the Moon and, to a lesser extent, the planets on the Earth's equatorial bulge.

5.1.8 Pseudo Earth Fixed

It is defined the same way as Earth Fixed reference frame. The difference is that polar motion rotation is not considered in this case.

5.1.9 Earth Fixed

The Earth fixed reference frame in use is the *IERS Terrestrial Reference Frame* (ITRF).

The zero longitude or IERS Reference Meridian (IRM), as well as the IERS Reference Pole (IRP), are maintained by the International Earth Rotation Service (IERS), based on a large number of observing stations, and define the IERS Terrestrial Reference Frame (ITRF).

5.1.10 Topocentric

Its z-axis coincides with the normal vector to the Earth's Reference Ellipsoid, positive towards zenith. The x-y plane is the plane orthogonal to the z-axis, and the x-axis and y-axis point positive, respectively, towards east and north.

5.1.11 LIF (Launch Inertial Frame)

The Launch Inertial Frame (LIF) is an Coordinate System whose Z axis and XY plane are the same as Z axis and XY plane of Earth Fixed Coordinate System. It is a user defined Coordinate System as the user needs to provide a longitude and a UTC time to define it. More precisely, X,Y and Z axes are defined as follows:

- X axis: is in the equatorial plane and points where a given user defined meridian was at a given user defined UTC time. The meridian is defined by the user specifying the value of the corresponding longitude (note that, if longitude = 0, the reference is the Greenwich meridian)

- Z axis: is in the direction of the angular velocity of the Earth (= towards North Pole)
- Y axis: complete the right-handed coordinate system

5.2 Satellite Reference Frames

Four levels of reference frames are used for attitude determination:

- The Satellite Orbital frame (SOF)
- Satellite Nominal reference frame (SNRF)
- Satellite reference frame (SRF)
- Instrument reference frame (IRF)

The SOF is used for the computation of the other satellite reference frames (see section 5.2.1 for the definition of this frame)

The SNRF is an ideal attitude model. The axis definition for this frame depends on the attitude model chosen for the satellite. Let's see two examples:

- Local Normal Pointing attitude (LNP), the z-axis is chosen in the direction of the satellite's zenith and the x-axis is defined in the direction of the satellite's inertial velocity vector (in True of Date).
- Yaw Steering Mode attitude (YSM): the z-axis is chosen in the direction of the satellite's zenith and the x-axis is defined in the direction of the satellite's velocity vector in the Earth Fixed CS.

A complete list of attitude models can be seen in section 8.1.

The SRF corresponds to the satellite actual (measured) attitude frame. It could be considered as the result of three consecutive rotations of the SNRF over three angles called ***mispointing angles***. The time derivative of those mispointing angles are called ***mispointing rates***.

Finally the IRF is a frame based on an instrument of the satellite. There exists one reference frame per instrument and it is used for location and looking direction from the instrument.

5.2.1 Satellite Orbital

It is a reference frame centred on the satellite and is defined by the Xs, Ys and Zs axes, which are specified relatively to the reference inertial reference frame, namely the True of Date.

The Zs axis points along the radial satellite direction vector, positive from the centre of the TOD reference frame towards the satellite, the Ys axis points along the transversal direction vector within the osculating orbital plane (i.e the plane defined by the position and velocity vectors of the satellite), orthogonal to the Zs axis and opposed to the direction of the orbital motion of the satellite. The Xs axis points towards the out-of-plane direction vector completing the right hand reference frame.

$$\vec{Z}=\frac{\vec{r}}{|\vec{r}|} \qquad \vec{X}=\frac{\vec{r}\wedge\vec{v}}{|\vec{r}\wedge\vec{v}|} \qquad \vec{Y}=\vec{Z}\wedge\vec{X}$$

where $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ are the unitary direction vectors in the (Xs, Ys, Zs) axes, and $\vec{r}$ and $\vec{v}$ are the position and velocity vectors of the satellite expressed in the inertial reference frame.

Next drawing depicts the Satellite Orbital frame:

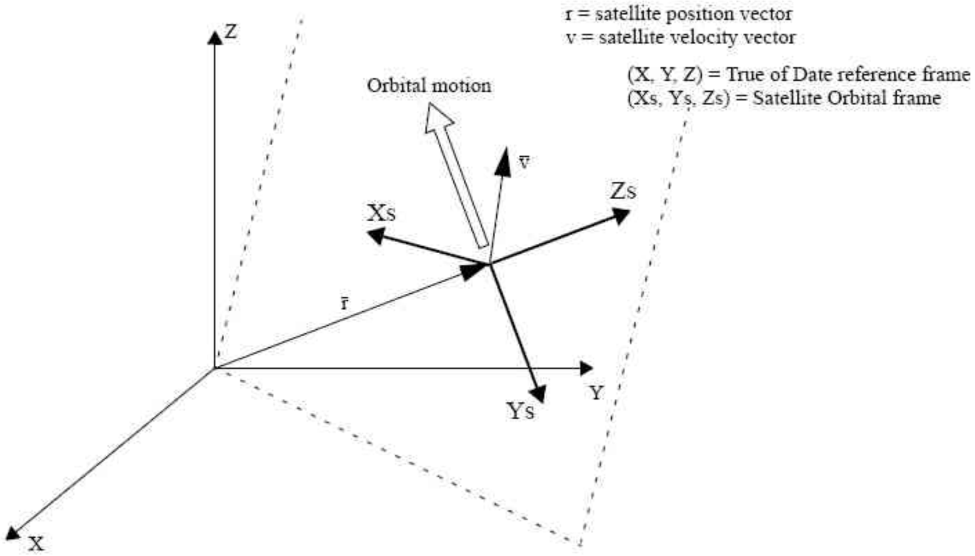
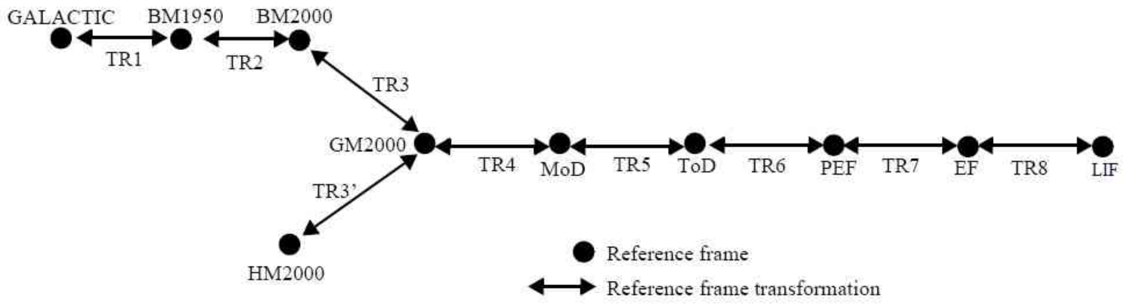


Figure 2: Satellite Orbital Frame

5.3 General Reference Frames Transformations

The following picture identifies the general reference frames transformations that are relevant for the Earth Observation missions.



Reference frames:

GALACTIC = Galactic CS (see section 5.1.1)
BM1950 = Barycentric Mean of 1950.0 (see section 5.1.2)
BM2000 = Barycentric Mean of 2000.0 (see section 5.1.3)
HM2000 = Heliocentric Mean of 2000.0 (see section 5.1.4)
GM2000 = Geocentric Mean of 2000.0 (see section 5.1.5)
MoD = Mean of Date (see section 5.1.6)
ToD = True of Date (see section 5.1.7)
PEF = Pseudo Earth Fixed (see section 5.1.8)
EF = Earth Fixed (see section 5.1.9)
LIF = Launch Inertial Frame (see section 5.1.11)

Transfromations:

TR1 = Galactic to Barycentric Mean of 1950 (see section 5.3.1)
TR2 = Barycentric 1950 to Barycentric 2000 (see section 5.3.2)
TR3 = Solar system barycentre to Earth centre translation (see section 5.3.3)
TR3' = Sun centre to Earth centre translation (see section 5.3.4)
TR4 = Precession (see section 5.3.5)
TR5 = Nutation (see section 5.3.6)
TR6 = Earth rotation + nutation term (see section 5.3.7)
TR7 = Polar motion rotation (see section 5.3.8)
TR8 = Earth rotation

Figure 3: General Reference Frames Transformations

Those transformation are described in the following sections.

Note that whenever a transformation is expressed as a sequence of rotations, the following expressions apply (the angle w is regarded positive):

$$R_X(w)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos w & \sin w\\ 0 & -\sin w & \cos w\end{bmatrix} \qquad R_Y(w)=\begin{bmatrix}\cos w & 0 & -\sin w\\ 0 & 1 & 0\\ \sin w & 0 & \cos w\end{bmatrix} \qquad R_Z(w)=\begin{bmatrix}\cos w & \sin w & 0\\ -\sin w & \cos w & 0\\ 0 & 0 & 1\end{bmatrix}$$

5.3.1 Galactic to Barycentric Mean of 1950

The following picture represents the galactic and the equatorial coordinate systems. The relationship between both systems are given by the equatorial coordinates of the galactic pole for the epoch 1950 and for by the position of the galactic centre.

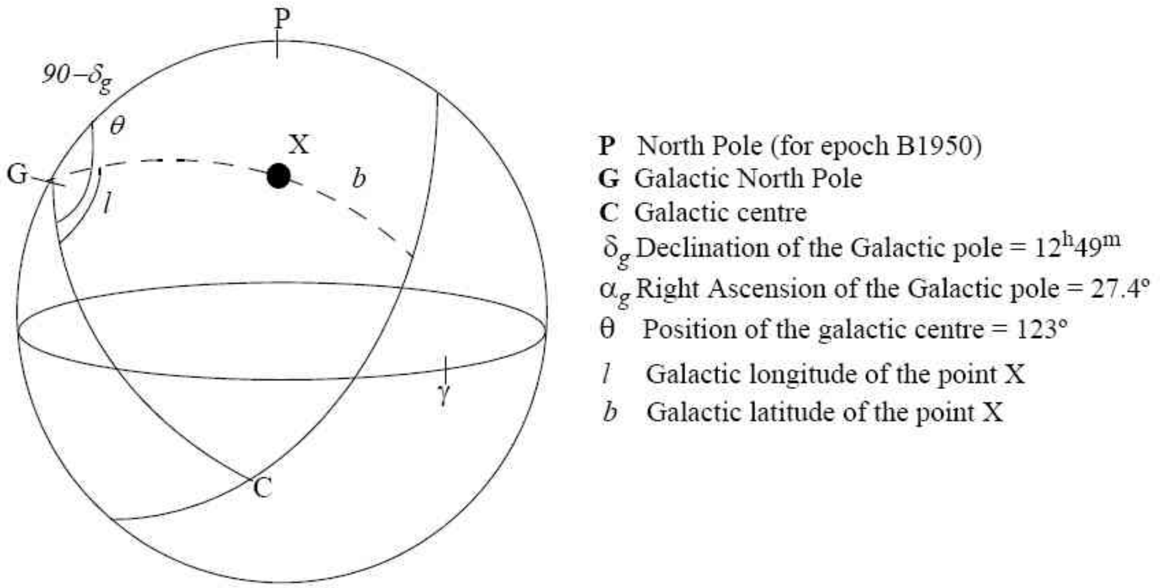


Figure 4: Galactic and Equatorial coordinates

In the figure, considering the spheric triangle GPX, the relationship between the galactic and equatorial coordinates can be established (see GREEN for further details)

$$\cos b \sin(\theta - l) = \cos\delta \sin(\alpha - \alpha_g)$$

$$\cos b \cos(\theta - l) = \cos\delta_g \sin\delta - \sin\delta_g \cos\delta \cos(\alpha - \alpha_g)$$

Taking into account the relations between spherical and cartesian coordinates, it is easy to derive the rotation matrix from Galactic to Barycentric B1950.0:

$$R_{(galactic \rightarrow B1950.0)} = \begin{bmatrix} -0{,}06698874 & 0{,}49272847 & -0{,}86760081 \\ -0{,}8727557659 & -0{,}45034696 & -0{,}1883746 \\ -0{,}4835389146 & 0{,}74458463 & 0{,}46019978 \end{bmatrix}$$

5.3.2 Barycentric Mean of 1950.0 to Barycentric Mean of 2000

The transformation from barycentric B1950.0 to barycentric J2000 includes the following processes:

1. Removal of the terms of elliptic aberration.
2. Rotation to the dynamical equinox of B1950.0
3. Correcting the proper motions for the equinox motion and the change in the value of precession
4. Changing from tropical to Julian centuries for the time scale of proper motions
5. Updating of positions to the epoch of J2000
6. Precession of positions and proper motions from B1950.0 to J2000.

For further details about this transformation, refer to:

- ALMAN05(B32)
- Astronomical and Astrophysical Journal 128, 263-267 (1983)

5.3.3 Barycentric Mean of 2000 to Geocentric Mean of 2000

The transformation from the Barycentric Mean of 2000 to the Geocentric Mean of 2000 reference frame is calculated with the following expressions (Figure 4):

$$\vec{r}_E = \vec{r}_B - \vec{r}_{B,\,Earth}$$

$$\vec{v}_E = \vec{v}_B - \vec{v}_{B,\,Earth}$$

where $\vec{r}_E$ and $\vec{v}_E$ are the position and velocity vectors in the Geocentric Mean of 2000 reference frame, $\vec{r}_B$ and $\vec{v}_B$ are the position and velocity vectors in the Barycentric Mean of 2000 reference frame, and $\vec{r}_{B,\,Earth}$ and $\vec{v}_{B,Earth}$ are the position and velocity vectors of the Earth in the Barycentric Mean of 2000 reference frame.

$\vec{r}_{B,\,Earth}$ and $\vec{v}_{B,Earth}$ are calculated according to BOWRING reference.

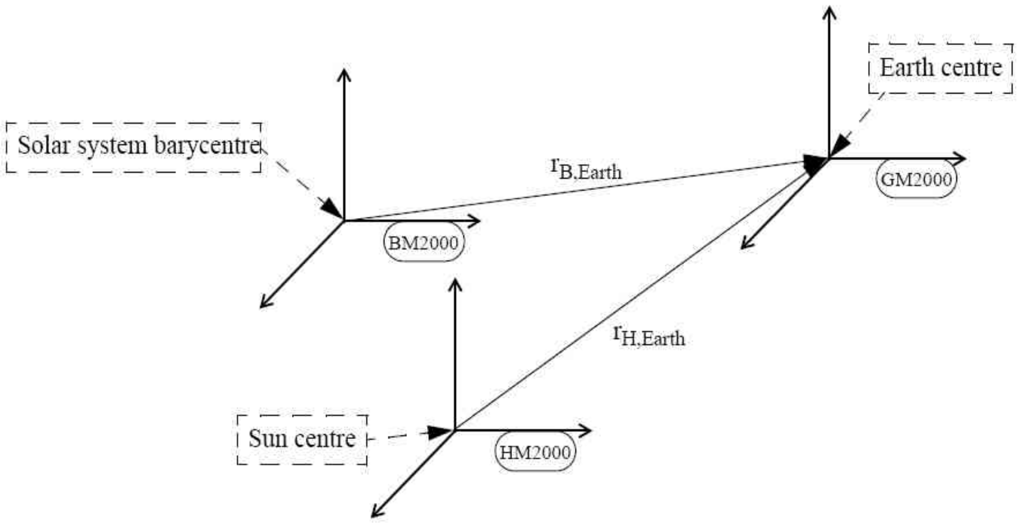


Figure 5: Transformations between BM2000, HM2000 and GM2000 reference frames

5.3.4 Heliocentric Mean of 2000 to Geocentric Mean of 2000

The transformation from the Heliocentric Mean of 2000 to the Geocentric Mean of 2000 reference frame is calculated with the following expressions (Figure 5):

$$\vec{r}_E = \vec{r}_H - \vec{r}_{H,\,Earth}$$

$$\vec{v}_E = \vec{v}_H - \vec{v}_{H,\,Earth}$$

where $\vec{r}_E$ and $\vec{v}_E$ are the position and velocity vectors in the Geocentric Mean of 2000 reference frame, $\vec{r}_H$ and $\vec{v}_H$ are the position and velocity vectors in the Heliocentric Mean of 2000 reference frame, and $\vec{r}_{H,\,Earth}$ and $\vec{v}_{H,Earth}$ are the position and velocity vectors of the Earth in the Heliocentric Mean of 2000 reference frame.

$\vec{r}_{H,\,Earth}$ and $\vec{v}_{H,Earth}$ are calculated according to BOWRING reference.

5.3.5 Geocentric Mean of 2000 to Mean of Date

The transformation from the Geocentric Mean of 2000 to the Mean of Date reference frame is performed with the following expression (Figure 6):

$$\vec{r}_m = R_z\left(-\frac{\pi}{2} - z\right) R_x(\theta) R_z\left(\frac{\pi}{2} - \zeta\right) \vec{r}_{J2000}$$

where $\vec{r}_m$ and $\vec{r}_{J2000}$ are the position vector in the Mean of Date and the Mean of 2000 reference frame, respectively.

The rotation angles of the precession model are calculated as follows (OAD_TIME reference):

$$\xi = 0.6406161T + 0.0000839T^2 + 0.0000050T^3 [deg]$$
$$z = 0.6406161T + 0.0003041T^2 + 0.0000051T^3 [deg]$$
$$\vartheta = 0.5567530T - 0.0001185T^2 - 0.0000116T^3 [deg]$$

where T is the TDB time expressed in the Julian centuries format (1 Julian century = 36525 days).

However, the precession motion is so slow that the UTC time can be used instead of the TDB time, and therefore T can be calculated from t, the UTC time expressed in the MJD2000 format, with the following expression:

T = (t - 0.5)/36525 [Julian centuries]

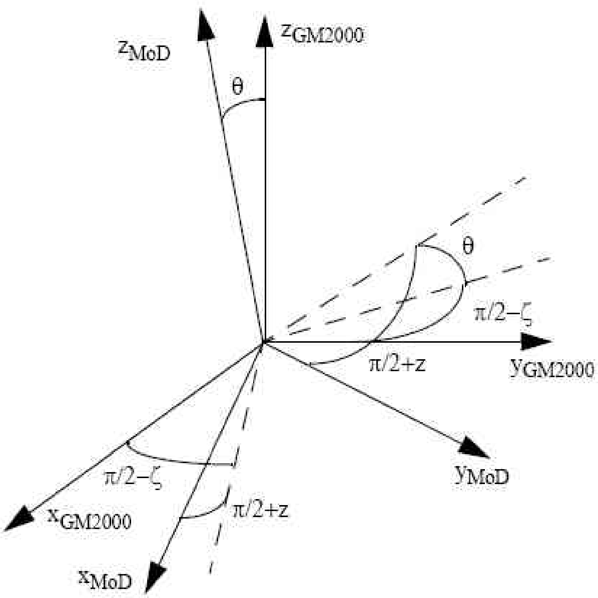


Figure 6: Transformation between GM200 and MoD reference frames

5.3.6 Mean of Date to True of Date

The transformation from the Mean of Date to the True of Date reference frame is performed with the following expression (Figure 7):

$$\vec{r}_t = R_z(-\delta\mu)R_x(-\delta\varepsilon)R_y(\delta\upsilon)\vec{r}_m$$

where $\vec{r}_t$ and $\vec{r}_m$ are, respectively, the position vector in the True of Date and the Mean of Date reference frame.

The rotation angles of the simplified nutation model are calculated with (OAD_TIME reference):

$$\delta\mu = \delta\psi\cos\varepsilon$$

$$\delta\upsilon = \delta\psi\sin\varepsilon$$

where ε is the obliquity of the ecliptic at the epoch J2000:

$\varepsilon = 23.439291$ [deg]

and $\delta\varepsilon$ and $\delta\psi$ is expressed by the *Wahr* model taking only the nine largest terms, and using UT1 instead of TDB as the time reference.

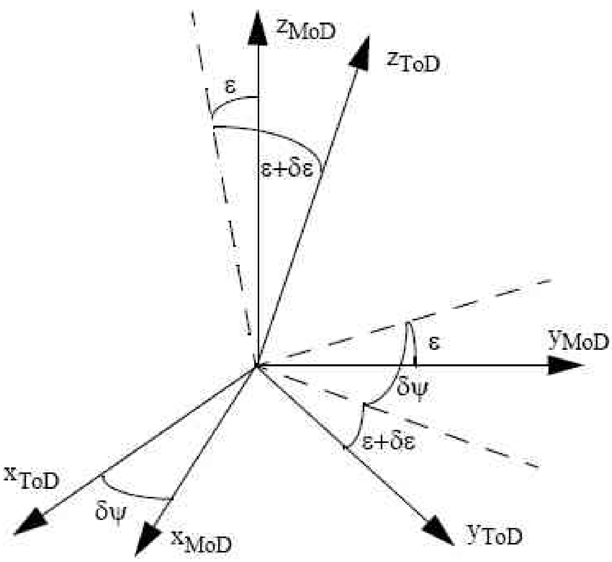


Figure 7: Transformation between MoD and ToD reference frames

5.3.7 True of Date to Pseudo Earth Fixed

The transformation from the True of Date to the Pseudo Earth fixed reference frame is performed with the following expression (Figure 8):

$$\vec{r}_{pe} = R_z(H)\vec{r}_t$$

where $\vec{r}_{pe}$ and $\vec{r}_t$ are, respectively, the position vector in the Pseudo Earth fixed and in the True of Date reference frames.

The ***Earth rotation angle H*** is the sum of the Greenwich sidereal angle and a small term from the nutation in the longitude of the equinox.

The Greenwich sidereal angle moves with the daily rotation of the Earth and is calculated with the Newcomb's formula according to international conventions as a third order polynomial, although the third order term will be neglected in our calculations.

The nutation term is calculated with the simplified nutation model (see section 5.1.7).

$$H = G + \delta\mu$$

$$G = 99.96779469 + 360.9856473662860T + 0.29079\,x\,10^{-12}\,T^2\,[deg]$$

where T is the UT1 time expressed in the MJD2000 format.

Note that the transformation from the Mean of Date to the Pseudo Earth fixed reference frame can be performed in one step being the δμ rotation term cancelled out:

$$\vec{r}_{pe} = R_z(G)\,R_x(-\delta\varepsilon)\,R_y(\delta\upsilon)\,\vec{r}_{qm}$$

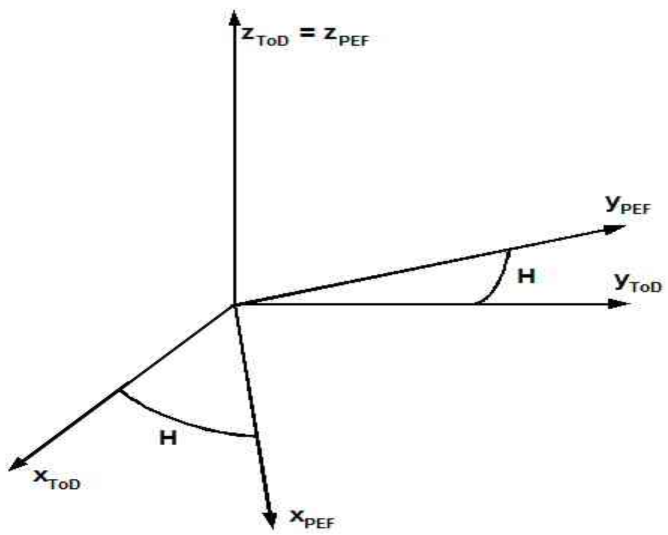


Figure 8: Transformation between ToD and PEF reference frames

5.3.8 Pseudo Earth Fixed to Earth Fixed

The transformation from the Pseudo Earth Fixed to the Earth fixed reference frame is performed with the following expression (Figure 9):

$$\vec{r}_{EF} = R_y(-X)\,R_x(-Y)\,\vec{r}_{PEF}$$

where $\vec{r}_{EF}$ and $\vec{r}_{PEF}$ are, respectively, the position vector in the Earth fixed and in the Pseudo Earth Fixed reference frames; X and Y are the polar motion parameters (measured and predicted by the IERS).

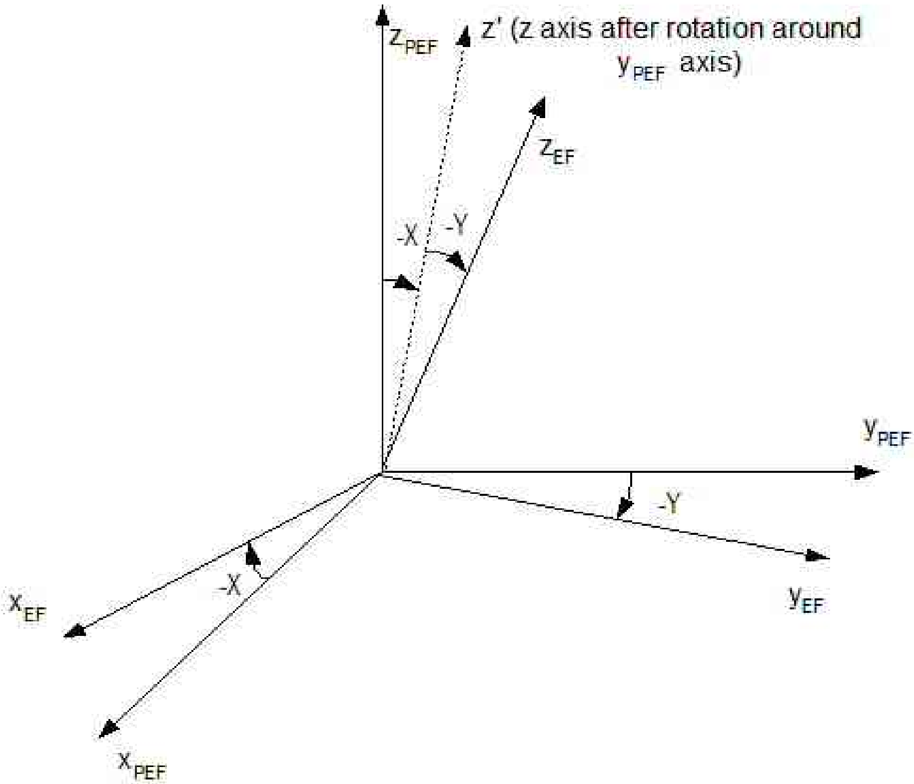


Figure 9: *Transformation between PEF and EF reference frames*

5.4 Satellite Reference Frames Transformations

There is not a general rule for transforming from one satellite reference frame to another. The attitude computation provides the transformation matrix from the satellite frame to an inertial reference frame. The following picture identifies the CFI-specific reference frames transformations that are relevant for the Earth Observation missions:

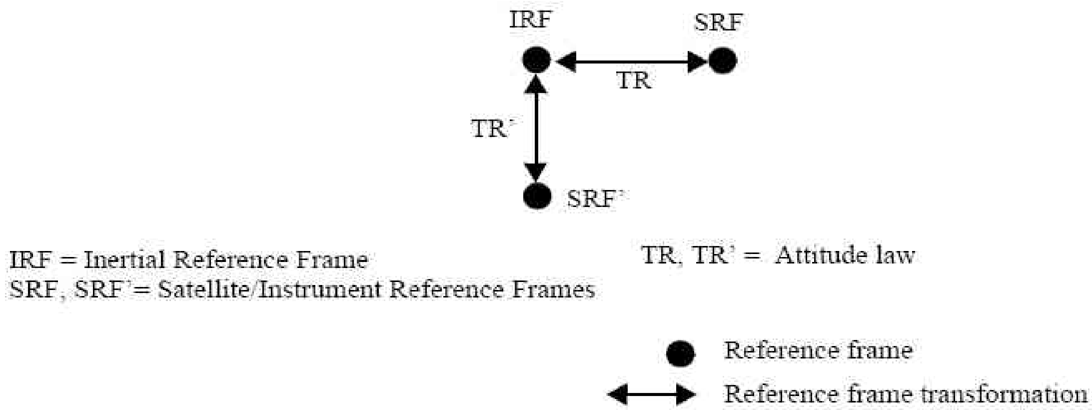


Figure 10: CFI-specific Reference Frames Transformations

6 ORBIT CHARACTERISATION

6.1 Orbit Definition

6.1.1 Sun-synchronous Orbit

The orbit is ***Sun-synchronous*** when the rate of change of the mean right ascension of the ascending node coincides with the motion of the mean Sun:

$$\dot{\Omega} = \dot{\bar{L}}_{sun}$$

which implies that the MLST of the ascending node is also constant. Its behaviour is graphically presented in Figure 11 (a).

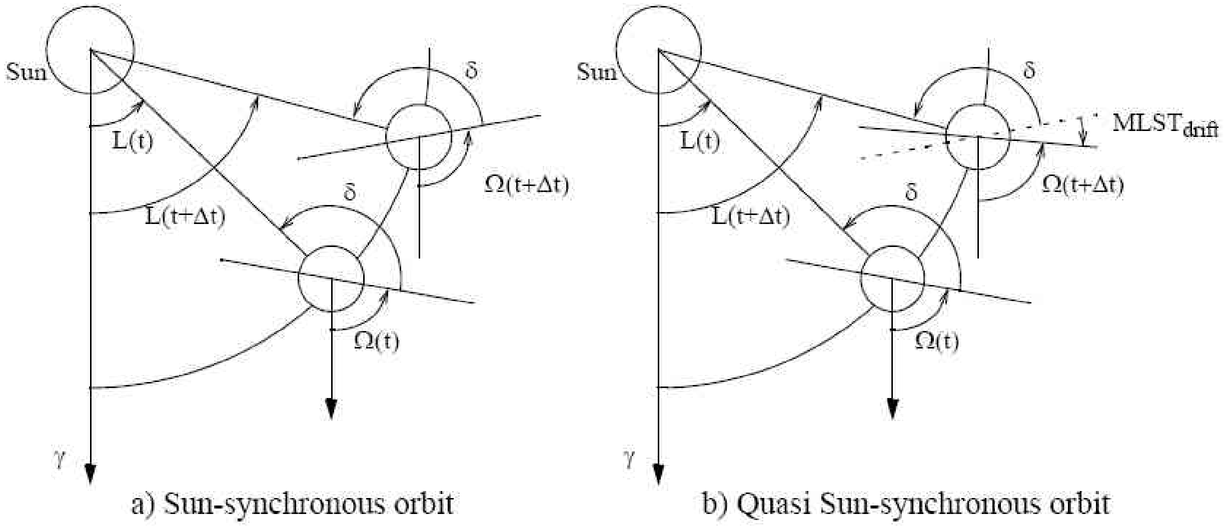


Figure 11: Sun-synchronous and quasi Sun-synchronous orbits descriptions

6.1.2 Quasi Sun-synchronous Orbit

The orbit is ***quasi Sun-synchronous*** when the rate of change of the mean right ascension of the ascending node is shifted from the motion of the mean Sun by a constant drift. It implies that the orbit line of nodes moves backward/forward with respect to the Sun-Earth LOS. The condition can be expressed mathematically in the following way:

$$\dot{\Omega} = \dot{\bar{L}}_{sun} + M\dot{L}ST_{drift}$$

The behaviour of a quasi Sun-synchronous orbit compared to that of a Sun-synchronous orbit is presented in Figure 11 (b).

6.1.3 Geo-synchronous Orbit

The orbit is Geo-synchronous when the ground track in the Earth Fixed reference frame repeats precisely after a constant integer number of orbits.

6.2 Orbit Types

6.2.1 Reference Orbit

The ***reference orbit*** consists of a scenario file, containing orbit information per repeat cycle change, i.e. the position and velocity vectors expressed in the Earth fixed reference frame, corresponding to the ascending node of that orbit and its associated time.

This state vector of the ascending node is calculated using the satellite-specific propagation mode, and imposing the conditions pertained the particular orbit definition.

6.2.2 Predicted Orbit

The predicted orbit consists of a set of satellite Cartesian state vectors that allow the computation of state vectors in the future with respect to the time in which the set has been generated.

6.2.3 Restituted Orbit

The restituted orbit consists of a set of consolidated satellite Cartesian state vectors that allow the computation of state vectors in the past with respect to the time in which the set has been generated.

6.2.4 TLE Orbit

The ***TLE orbit*** consists of two 69-character lines of data. The TLE contains mean keplerian elements for a given epoch (More info about TLE format can be found in CELES).

6.3 Orbit Propagation Definition

To calculate the state vector at any point in the orbit, it is sufficient to have the orbital data (a state vector, or keplerian elements) at a given time, and then propagate that initial state vector to the required time using an orbit propagation model.

That initial orbital data can come from different sources (see section 6.2) and depending on the type of orbit and the satellite mission, there are different requirements on the accuracy of the position and velocity vectors of that initial state.

6.4 Orbit Propagation Models

The propagation models must incorporate an initialisation mode. It basically starts with:

- a TLE providing the mean keplerian elements at a given epoch.
- an initial cartesian state vector expressed in the Earth fixed reference frame at a given time, supplied externally (see section 6.2), to calculate the time and the state vector of the true ascending node in the Earth fixed reference frame (i.e. $z_{AN} = 0$ and $\dot{z}_{AN} > 0$).

The initialisation mode implements an iterative algorithm which is based upon a propagation mode.

6.4.1 Mean Keplerian Orbit Propagator

6.4.1.1 Simulation mode

The simulation mode is one of reduced accuracy. In this case only the zonal (i.e. latitude independent) of the geoid J_2, J_2^2, J_3 and J_4 are used to calculate the secular perturbations of the *mean*[4] Kepler elements, and the zonal harmonic J2 is used to calculate the short periodic perturbations to transform the mean Kepler elements to the osculating Kepler elements.

This mode is based on the equations derived in LIU_ALFORD reference.

6.4.2 Precise Orbit Propagator

This model consists on a numerical propagator that integrates the movement equations using a Runge-Kutta algorithm of 8th order. This propagator is expected to produce more precise results than the other models. The main characteristics of the model are:

Characteristics	
Central Body	Earth
Integrator algorithm	Runge-Kutta 8 with a fixed integrator step defined by the user.
Perturbation models	• Gravity • Atmosphere • Third Body - Sun • Third Body - Moon • Solar Radiation Pressure
Gravity Model	EGM-96
Gravity Model Coefficients (Zonals x Tesserals)	36x36
Atmosphere model	MSIS-E-90
Solar Activity	Constant user-defined value
Geomagnetic Activities	Constant user-defined value
Sun Ephemerides	Analytical
Moon Ephemerides	Analytical
Aerodynamic Drag – Area	Constant user-defined value
Aerodynamic Drag – Drag Coeffcient (C_D)	Constant user-defined value
Solar Radiation Pressure – Area	Constant user-defined value
Solar Radiation Pressure – SRP Coeffcient (C_P)	Constant user-defined value

4 Averaged with respect to the osculating mean anomaly over 2π.

6.4.3 TLE Propagator

This model propagates the state vector using the NASA/NORAD "two line elements" and the SGP4 propagation model. SGP4 algorithm was designed by the NASA/NORAD for near Earth Satellites (nodal period less than 225 minutes). The SGP4 theory uses an Earth gravitational field through zonal terms J2, J3 and J4 and a power density function for the atmospheric model (assuming a non-rotating spherical model).

7 PARAMETERS

7.1 Orbit Parameters

7.1.1 Cartesian State Vector

It comprises the cartesian components of the position $\vec{r}_{SC}$, velocity $\vec{v}_{SC}$ and acceleration $\vec{a}_{SC}$ vectors of the satellite expressed in a specified reference frame (typically the Earth fixed reference frame) at a given epoch.

7.1.2 Orbit Radius, Velocity Magnitude and Components

The satellite ***orbit radius*** is the module of the satellite position vector $\vec{r}_{SC}$:

$$R=|\vec{r}_{SC}|$$

The ***velocity magnitude*** is the module of the satellite velocity vector $\vec{v}_{SC}$:

$$V=|\vec{v}_{SC}|$$

The satellite ***velocity*** vector when is expressed in the True of Date reference frame can be split into two ***components***:

- Radial component: $\vec{v}_r=\vec{v}_{SC}\cdot\vec{Z}$
- Transversal component: $\vec{v}_t=-\vec{v}_{SC}\cdot\vec{Y}$

where $\vec{Y}$ and $\vec{Z}$ are the direction vectors of the Satellite Reference frame (see section 5.2.1).

7.1.3 Osculating Kepler State Vector

The ***osculating Kepler elements*** are related to the cartesian state vector, at the corresponding epoch, expressed in the True of Date reference frame.

The six Kepler elements are:

- Semi-major axis (a)
- Eccentricity (e)
- Inclination (i)
- Argument of perigee (ω)
- Mean anomaly (M)
- Right ascension of the ascending node (Ω)

Other auxiliary elements are:

- Eccentric anomaly (E)
- True anomaly (υ)
- True latitude (α)
- Mean latitude (β)

The relationships between these auxiliary elements and the six Kepler elements are:

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\upsilon}{2}$$

$$M = E - e\sin E \text{ (Kepler's equation)}$$

$$\alpha = \omega + \nu$$

$$\beta = \omega + M$$

7.1.4 Mean Kepler State Vector

The osculating six Kepler elements in the True of Date reference frame can be averaged with respect to the mean anomaly over 2π , to obtain the mean Kepler elements:

$$\bar{a}, \bar{e}, \bar{i}, \bar{\omega}, \bar{\Omega}, \bar{M}$$

7.1.5 Equinoctial State Vector

The osculating Kepler elements are usually replaced by the equivalent osculating ***equinoctial elements*** for quasi-equatorial and quasi-circular orbits:

- $x_1 = a$
- $x_2 = e_x = e\cos(\Omega+\omega)$
- $x_3 = e_y = e\sin(\Omega+\omega)$
- $x_4 = i_x = +2\sin(i/2)\sin(\Omega)$
- $x_5 = i_y = -2\sin(i/2)\cos(\Omega)$
- $x_6 = \Omega + \omega + M$

7.1.6 Ascending Node, Ascending Node Time, Nodal Period, Absolute Orbit Number

The ***ascending node*** of an orbit is the intersection of that orbit, when the satellite goes from the southern to the northern hemisphere, with the x-y plane of the Earth fixed reference frame.

The ***ANX time*** is the UTC time of that ascending node.

The ***relative time*** with respect to the ANX time is the time elapsed since that ascending node till the current position within the orbit.

The ***nodal period*** of an orbit is the interval of time between two consecutive ascending nodes.

The Launch orbit from Kourou is regarded as ***absolute orbit number*** zero. From then on, each time a new ascending node is crossed the absolute orbit number is incremented by one.

7.1.7 Mean Local Solar Time Drift

The ***Mean Local Solar Time drift*** is the difference in angular velocity between the rate of change of the mean right ascension node and the motion of the mean Sun. This constant drift produces an increasing gap between the MLST of the ascending node and the angle measured from the line of nodes and the vernal equinox direction (see section 6.1.2). For a Sun-synchronous orbit, the MLST drift is zero.

The relationship between MLST of subsequent days is the following:

$$\mathrm{MLST}_{\mathrm{dayN}} = \mathrm{MLST}_{\mathrm{day(N-1)}} + \mathrm{MLST}_{\mathrm{drift}}$$

7.1.8 Repeat Cycle and Cycle Length

In the geo/helio-synchronous orbits, the ground track repeats precisely after a constant integer number of orbits and a constant duration. The duration in days of that period is called the ***repeat cycle***, whereas the corresponding number of orbits is called the ***cycle length***.

The repeat cycle of a Sun-synchronous orbit is an integer number of days, while it is not an integer number when considering a non Sun-synchronous orbit. Thus, the orbit information contained within a scenario file comprises an integer repeat cycle plus a drift on it, to cope with non Sun-synchronous orbits. The true repeat cycle shall result from the following:

$$\mathrm{TrueRepeatCycle} = \mathrm{RepeatCycle}(1 + \mathrm{MLSTdrift})$$

7.1.9 Sub-satellite Point, Satellite Nadir and Ground Track

The ***subsatellite point (SSP)*** is the normal projection of the position of the satellite in the orbit on to the surface of the Earth´s Reference Ellipsoid. It is also referred as ***nadir***.

The trace made by the subsatellite point on the surface of the Earth's Reference Ellipsoid due to the motion of the satellite along its orbit is called the ***ground track***.

7.1.10 Mean Local Solar Time and True Local Solar Time

7.1.10.1 Mean Local Solar Time

The ***Mean Local Solar Time (MLST)*** is the difference between the right ascension of the selected point in the orbit RA and the *mean longitude of the Sun* L, expressed in hours.

$$\mathrm{MLST} = (\mathrm{RA} - \mathrm{L} + \pi)\frac{24}{2\pi} \quad \text{[hours]}$$

The mean longitude $\bar{L}$ of the Sun represents the motion of the ***mean Sun*** and is given, in the Mean of Date reference frame, by (FLANDERN reference):

$$\bar{L} = 280.46592 + 0.9856473516(t - 0.5) \text{ [deg]}$$

where *t* is the UT1 time expressed in the MJD2000 format.

The motion of the mean Sun has a constant mean longitude rate, namely $\dot{L}$ = 0.9856473516 [deg/s].

7.1.10.2 True Local Solar Time

The ***True Local Solar Time (TLST)*** is the difference between the right ascension of the selected point in the orbit RA and the right ascension of the Sun RA_{SUN}, expressed in hours.

$$TLST = (RA - RA_{Sun} + \pi)\frac{24}{2\pi} \quad \text{[hours]}$$

The RA_{Sun} is calculated, in the Mean of Date reference frame, according to FLANDERN reference.

Mean and True Local Solar Time are normally expressed in hours considering the equivalence existing between hours and degrees; i.e. the Earth completes a complete revolution with respect to the Sun (360 degrees) in one day (24 hours).

7.1.11 Phase and Cycle

The phase is considered to be a portion of the mission characterised by a ground track pattern different from the previous and following. Each time a change of repeat cycle period is applied, a new phase starts. The decision of starting a new phase is performed by the mission management.

A cycle is defined as a full completion of the repeat period. A cycle starts by definition on an ascending node crossing closest to the Greenwich Meridian.

7.1.12 Absolute and Relative Orbit Number

The absolute orbit number considers the orbits elapsed since the first ascending node crossing after launch.

The relative orbit number is a count of orbits from 1 to the number of orbits contained in a repeat cycle. The relative orbit number 1 corresponds to the orbit whose ascending node crossing is closest to the Greenwich Meridian (eastwards). The relative orbit number is incremented in parallel to the absolute orbit number up to the cycle length, when it is reset and the cycle number is incremented by one.

When an orbit change is introduced, the relative orbit number of the new orbit is calculated such that the definition of the relative orbit number 1 is kept in the new repeat cycle.

7.1.13 Track Number

The track number is a count of orbits from 1 to the number of orbits contained in a repeat cycle. The track number 1 corresponds to the orbit whose ascending node crossing is closest to the Greenwich Meridian (eastwards). Two subsequent track numbers are those which have the nearest longitude of its ascending node crossing. Track number counter is incremented eastwards.

Track number 1 and relative orbit number 1 correspond to the same orbit. Furthermore, it exists a one-to-one relationship between track and relative orbit numbers within a repeat cycle.

7.1.14 Spacecraft Midnight

The Spacecraft Midnight (SMX) is the time just halfway the nadir day → night transition and the nadir night → day transition. Such transitions are times at which the Sun Zenith Angle (SZA, angle satellite-nadir-sun) is 90 deg. In the day → night transition, the SZA is increasing (i.e. there is a transition from SZA<90 to SZA>90). In the night → day transition the SZA is decreasing (i.e. there is a transition from SZA>90 to SZA<90).

7.2 Attitude Coordinate Systems Parameters

7.2.1 Attitude determination parameters

There are different ways for providing the attitude parameters in order to establish the transformations between the satellite reference frames:

1. Attitude Mispointing Angles:

The transformation from one satellite reference frame to another is accomplished by three consecutive rotations over the angles pitch η, roll ξ and yaw ζ according to the Euler convention defined in section 7.7.

The time derivative of those angles are the ***pitch, roll and yaw rates***.

Both those angles and their rates are a function of the selected attitude control mode (see attitude control section particular to each satellite).

Usually these angles are used for transforming from the satellite orbital frame to the satellite nominal attitude frame. Frequently there are superimposed on them a set of ***mispointing angles*** that make the Satellite Nominal Attitude Reference frame transform to the Satellite Attitude Reference frame.

The mispointing angles are expressed as three components, namely pitch $\Delta\eta$, roll $\Delta\xi$, and yaw $\Delta\zeta$.

The time derivative of those mispointing angles are the ***mispointing rates***.

2. Attitude Quaternions:

The previous transformations could be given via quaternions (also called Euler symmetric parameters) instead of angles.

Quaternions are base on Euler's theorem that given two coordinate systems, there is one invariant axis (***e***) along which measurements are the same in both coordinate systems and that is possible to move from one system to the other through a rotation (β) about the axis ***e***. According to this theorem, the quaternions is defined as:

$$\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} \mathbf{e}\sin\frac{\beta}{2} \\ \cos\frac{\beta}{2} \end{bmatrix} = \begin{bmatrix} e_x \sin\frac{\beta}{2} \\ e_y \sin\frac{\beta}{2} \\ e_z \sin\frac{\beta}{2} \\ \cos\frac{\beta}{2} \end{bmatrix}$$

A rotation matrix (direction cosine matrix) can be expressed in term of the quaternion parameters as follows:

$$R = \begin{bmatrix} (q_1^2 - q_2^2 - q_3^2 + q_4^2) & 2(q_1 q_2 + q_3 q_4) & 2(q_1 q_3 - q_2 q_4) \\ 2(q_1 q_2 - q_3 q_4) & (-q_1^2 + q_2^2 - q_3^2 + q_4^2) & 2(q_2 q_3 + q_1 q_4) \\ 2(q_1 q_3 + q_2 q_4) & 2(q_2 q_3 - q_1 q_4) & (-q_1^2 - q_2^2 + q_3^2 + q_4^2) \end{bmatrix}$$

There are for possible solutions for getting the quaternion from the rotation matrix:

$$\mathbf{Q}_1 = \begin{bmatrix} \frac{\sqrt{1 + R_{11} - R_{22} - R_{33}}}{2} \\ \frac{1}{4Q_1}(R_{12} + R_{21}) \\ \frac{1}{4Q_1}(R_{13} + R_{31}) \\ \frac{1}{4Q_1}(R_{23} - R_{32}) \end{bmatrix} \quad \mathbf{Q}_2 = \begin{bmatrix} \frac{1}{4Q_2}(R_{12} + R_{21}) \\ \frac{\sqrt{1 - R_{11} + R_{22} - R_{33}}}{2} \\ \frac{1}{4Q_2}(R_{23} + R_{32}) \\ \frac{1}{4Q_2}(R_{31} - R_{13}) \end{bmatrix} \quad \mathbf{Q}_3 = \begin{bmatrix} \frac{1}{4Q_3}(R_{13} + R_{31}) \\ \frac{1}{4Q_3}(R_{23} + R_{32}) \\ \frac{\sqrt{1 - R_{11} + R_{22} - R_{33}}}{2} \\ \frac{1}{4Q_3}(R_{12} - R_{21}) \end{bmatrix} \quad \mathbf{Q}_4 = \begin{bmatrix} \frac{1}{4Q_4}(R_{23} - R_{32}) \\ \frac{1}{4Q_4}(R_{31} - R_{13}) \\ \frac{1}{4Q_4}(R_{12} - R_{21}) \\ \frac{\sqrt{1 - R_{11} + R_{22} - R_{33}}}{2} \end{bmatrix}$$

The EOCFI returns the weighted mean of the four possible solutions (with q_4 as real part of the quaternion):

$$\mathbf{q} = \begin{bmatrix} Q_1^2 \\ Q_2^2 \\ Q_3^2 \\ Q_4^2 \end{bmatrix} = \begin{bmatrix} \frac{1 + R_{11} - R_{22} - R_{33} + R_{12} + R_{21} + R_{13} + R_{31} + R_{23} - R_{32}]}{4} \\ \frac{R_{12} + R_{21} + 1 - R_{11} + R_{22} - R_{33} + R_{23} + R_{32} + R_{31} - R_{13}}{4} \\ \frac{R_{13} + R_{31} + R_{23} + R_{32} + 1 - R_{11} + R_{22} - R_{33} + R_{12} - R_{21}}{4} \\ \frac{R_{23} - R_{32} + R_{31} - R_{13} + R_{12} - R_{21} + 1 - R_{11} + R_{22} - R_{33}}{4} \end{bmatrix}$$

3. AOCS Rotation Amplitudes

The AOCS rotation amplitudes are the three constants Cx, Cy and Cz that define the transformation from the Satellite Nominal Attitude to the Satellite Attitude Reference frame according to the selected attitude control mode (see attitude control section particular to each satellite).

7.2.2 Satellite Centered Direction

The parameters that define a direction in a Satellite Reference frame are the satellite related azimuth (Az) and the satellite related elevation (El):

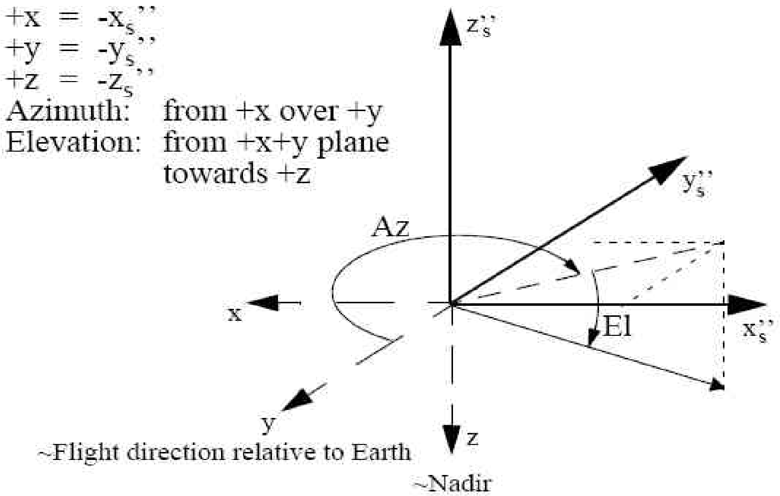


Figure 12: Satellite centred direction

7.3 Earth-related Parameters

Note that altitude refers always to geodetic altitude except when the contrary is explicitly said.

7.3.1 Geodetic Position

The geodetic coordinates of a point, related to the Earth´s Reference Ellipsoid, are the ***geocentric longitude*** λ, ***geodetic latitude*** φ, and ***geodetic altitude*** h, represented in Figure 13.

The ***geocentric latitude*** φ', ***geocentric radius*** ρ and the ***geocentric distance*** d are also represented in Figure 13 .

The parameters **a**, **e** and **f**, i.e. the semi-major axis, the first eccentricity and the flattening of the Earth's Reference Ellipsoid (see section 8.2.2), define the equations that express these other parameters.

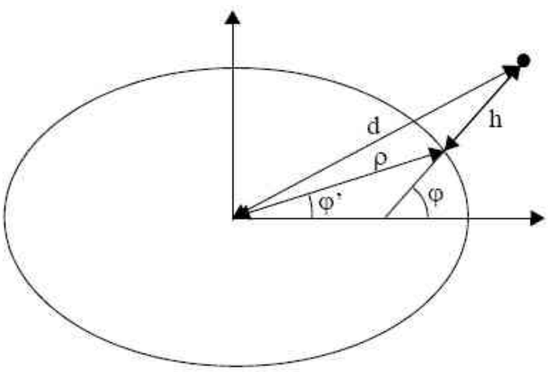


Figure 13: Geodetic position

The geocentric latitude φ’ and the geodetic latitude φ are related by the expression:

$$\tan(\varphi) = \frac{1}{(1-f)^2} \tan(\varphi)'$$

The geocentric radius ρ is calculated with:

$$\rho = \frac{a\sqrt{1-e^2}}{\sqrt{1-e^2 \cos^2 \varphi'}}$$

The relationship between the cartesian coordinates of a point and its geodetic coordinates is:

$$x = (N+h)\cos(\varphi)\cos(\lambda)$$
$$x = (N+h)\cos(\varphi)\sin(\lambda)$$
$$x = [(1-e^2)N + h]\sin(\varphi)$$

where N is the ***East-West radius of curvature***:

$$N = \frac{a}{\sqrt{1-e^2 \sin^2 \varphi}}$$

The inverse transformation, from the cartesian to the geodetic coordinates, cannot be performed analytically. The iterative method that will be used will be initialized according to (BOWRING reference).

The normal projection of a point on the surface of the Earth’s Reference Ellipsoid is called ***Nadir***, and when that point corresponds to the position of the satellite, the projection is called subsatellite point.

Another important radius of curvature is M, the ***North-South radius of curvature***:

$$M = \frac{a(1-e^2)}{\sqrt{(1-e^2 \sin^2 \varphi)^3}}$$

The ***radius of curvature*** in any selected direction R_{Az} can be calculated with the expression:

$$\frac{1}{R_{Az}} = \frac{\cos^2(Az)}{M} + \frac{\sin^2(Az)}{N}$$

where Az is the angle of the selected direction expressed in the Topocentric reference frame.

The ***satellite centred aspect angle*** $\alpha_{s/c}$ is the angle measured at the satellite between the *geometric direction*[5] *from the satellite to the subsatellite point and the geometric direction from the satellite to the centre of the Earth.*

The ***geocentric aspect angle*** α_g is the angle measured at the centre of the Earth between the geometric direction from the Earth centre to the subsatellite point and the geometric direction from the Earth centre to the satellite.

The ***subsatellite point centred aspect angle*** α_{ssp} is the angle measured at the subsatellite point between the geometric direction from the subsatellite point to the satellite and the geometric direction from the subsatellite point to the centre of the Earth.

The geodesic ***distance*** or ***ground range*** between two points that lay on an ellipsoid is by definition the minimum distance between those two points measured over that ellipsoid.

The ***velocity*** $\vec{v}_E$ and $\vec{a}_E$ ***acceleration relative to the Earth***, i.e the Earth's Reference Ellipsoid, of a point that lays on its surface can be split into different components.

- Northward component = $\vec{v}_E \cdot \vec{N}$ or $\vec{a}_E \cdot \vec{N}$
- Eastward component = $\vec{v}_E \cdot \vec{E}$ or $\vec{a}_E \cdot \vec{E}$
- Ground track tangential component = $\vec{v}_E \cdot \dot{t} = v_E$ or $\vec{a}_E \cdot \dot{t}$
- Magnitude = $v_E = |\vec{v}_E|$ or $a_E = |\vec{a}_E|$
- Azimuth = the azimuth of the $\vec{v}_E$ and $\vec{a}_E$ vectors measured in the Topocentric reference frame

where $\vec{N}$ and $\vec{E}$ are the north and east direction axes of the Topocentric reference frame centred on that point, and $\dot{t}$ is the unitary vector tangent to the ground track at that point.

5 The geometric direction is defined by the straight line that connects the initial and the final point.

7.3.2 Earth Centered Direction

The parameters that define a direction from the centre of the Earth to a point in the Mean of Date reference frame are the right ascension (α) and the declination (δ), shown in next figure:

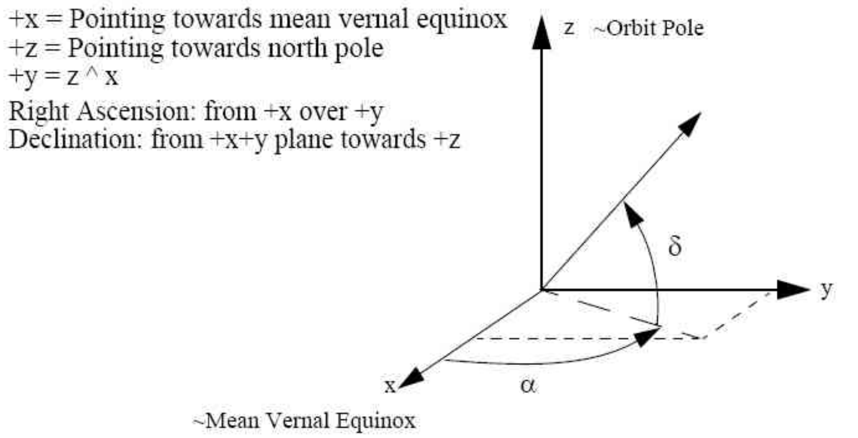


Figure 14: Earth centred direction

7.3.3 Topocentric Direction

The parameters that define a direction in the Topocentric reference frame are the topocentric azimuth (Az) and the topocentric elevation (El), represented in the next drawing:

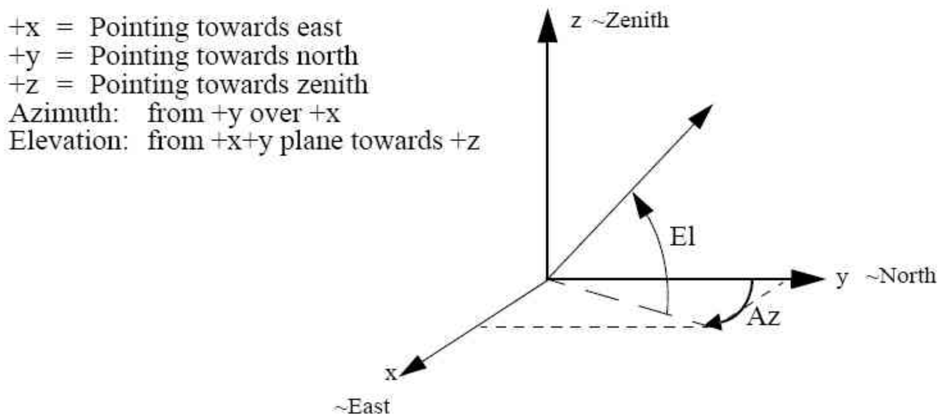


Figure 15: Topocentric direction

7.4 Ground Station Parameters

7.4.1 Ground Station Location

The location of a ***Ground Station*** is defined by its geodetic parameters: i.e. geocentric longitude λ, geodetic latitude φ, and geodetic altitude h with respect to the Earth's Reference Ellipsoid.

7.4.2 Ground Station Visibility

The visibility of a point from a Ground Station is limited by the ***minimum link elevation*** at which that point must be in order for the link between that Ground Station and that point to be established.

That minimum topocentric elevation is expressed in the Topocentric reference frame centred at that Ground Station (see section 7.3.3), and although it is ideally a constant, in fact a real Ground Station usually has a physical mask that makes the minimum topocentric elevation be a function of the topocentric azimuth.

7.5 Target Parameters

7.5.1 Moving and Earth-fixed Targets

A ***target*** $\vec{r}_t$ is a point that is observed from the satellite and that satisfies certain conditions.

The ***look direction***, or ***line of sight*** (LOS), $\vec{u}_0$ is the light direction, at the satellite, of the path followed by the light in its travel from the target to the satellite.

If the target moves with respect to the Earth, as a result of a change in the satellite position or a change in the look direction, it is called the ***moving target***.

If the target is fixed with respect to the Earth, which implies that if the satellite position changes then the look direction has to change in the precise way to keep looking to that particular point fixed to the Earth, it is called the ***Earth fixed target***.

In other words, the velocity of the moving target is the result of the motion of the satellite and the change in the look direction, or in the conditions that define it, with time. On the other hand, the velocity of the Earth fixed target is only a function of the position of that point with respect to the Earth's Reference Ellipsoid and the rotation of the Earth fixed reference frame.

7.5.2 Location Parameters

The location of a target is defined by its geodetic parameters: i.e. geocentric longitude λ, geodetic latitude φ, and geodetic altitude h with respect to the Earth's Reference Ellipsoid, although it also can be defined by its cartesian position vector (x, y, z) expressed in the Earth fixed reference frame.

7.6 Sun and Moon Parameters

The ***Sun semi-diameter*** D_{Sun} is the apparent semi-diameter of the Sun, expressed in radians, as seen from the satellite, and is calculated with the equation:

$$D_{Sun} = \frac{d_{Sun}}{R_{Sun-SC}}$$

where $d_{Sun} = 6.96 \times 10^8$ [m] is the semi-diameter of the Sun, and $R_{Sun\text{-}S/C}$ is the geometric distance between the satellite and the Sun centre.

The ***Moon semi-diameter*** D_{Moon} is the apparent semi-diameter of the Moon, expressed in degrees, as seen from the satellite, and is calculated with the equation:

$$D_{Moon} = \frac{d_{Moon}}{R_{Moon-SC}}$$

where d_{Moon} = 1738000 [m] is the semi-diameter of the Moon, and $R_{Moon\text{-}S/C}$ is the geometric distance between the satellite and the Moon centre.

The ***area of the Moon lit by the Sun*** $A_{Moon\text{-}Sun}$ is calculated with the expression:

$$A_{Moon-Sun} = \frac{1+\cos\theta_{Sun-Moon-SC}}{2}$$

where $\theta_{Sun\text{-}Moon\text{-}S/C}$ is the angle measured at the centre of the Moon between the geometric direction from the centre of the Moon to the centre of the Sun and the geometric direction from the centre of the Moon to the satellite.

If $A_{Moon\text{-}Sun} = 0$ it is a new Moon, and if $A_{Moon\text{-}Sun} = 1$ it is a full Moon

The ***satellite eclipse flag*** indicates whether or not the path followed by the light from the centre of the Sun to the satellite intersects the Earth´s Reference Ellipsoid. It is equivalent to the ***satellite to Sun visibility flag***.

The ***satellite to Moon visibility flag*** indicates whether or not the path followed by the light from the centre of the Moon to the satellite intersects the Earth´s Reference Ellipsoid.

The ***target to Sun visibility flag*** indicates whether or not the path followed by the light from the centre of the Sun to the target intersects the Earth´s Reference Ellipsoid.

7.7 Euler angles

The Earth Observation CFI applies the following convention when using Euler angles to rotate one reference frame to another.

The rotated reference frame (X's, Y's, Z's) is obtained by applying three consecutive rotations to the original reference frame:

1. Rotation around -Ys over a roll angle η
2. Rotation around -X^1s (i.e the rotated Xs) over a pitch angle ξ
3. Rotation around +Z^2s (i.e the rotated Z^1s) over a yaw angle ζ.

Next drawing depicts the three rotations:

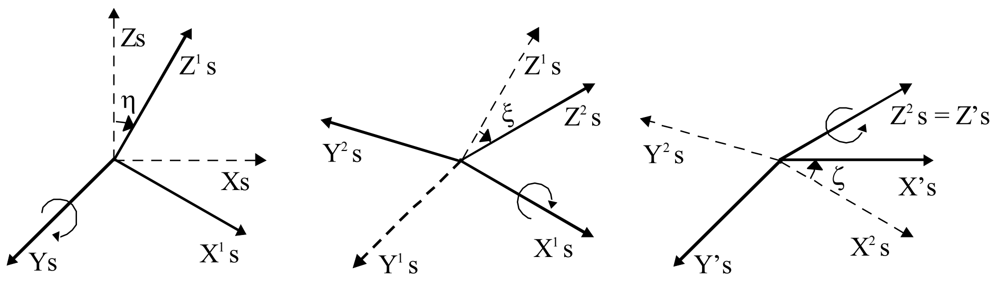


Figure 16: Euler Angles

8 MODELS

8.1 Attitude Modes

The Earth Observation CFI Software supports the following three axes stabilized Attitude Modes:

- Generic Pointing Mode (GPM);
- Yaw Steering Mode (YSM);
- Local Normal Pointing (LNP);
- Zero-Doppler Pointing (ZDOP);
- Geocentric Pointing (GP).

The GPM allows the user to define the Satellite Nominal Reference Frame (SNRF, see section 5.2) in the most generic way by selecting:

- The PRIMARY axis and direction (+/-X, +/-Y, +/- Z);
- The PRIMARY axis target (see list below);
- The SECONDARY axis and direction (+/-X, +/-Y, +/- Z, different from primary axis);
- The SECONDARY axis target (see list below).

The SNRF is defined as follows:

- The primary axis (with direction) is aligned to the primary axis target;
- The secondary axis (with direction) is aligned to the vector obtained by computing the cross-product between primary axis (positive direction) and the secondary target axis;
- The tertiary axis completes the right-hand frame together with primary and secondary axes.

For example, with reference to Table XX, SNRF in Yaw Steering mode is calculated as follows:

- The negative Z axis is aligned with the satellite position vector in the Earth Fixed reference system;
- The positive X axis is aligned with the cross product of Z axis and the velocity in the Earth Fixed reference system;
- Y axis is computed according to right-hand rule.

There are many possible choices in the definition of the axis target, for example:

- Sun pointing
- Moon pointing
- Earth pointing
- Nadir pointing
- Inertial velocity pointing
- Earth Fixed velocity pointing
- Inertial target pointing
- Earth Fixed target pointing
- Earth Fixed Satellite position

YSM, GP, LNP and ZDOP modes are special cases of Generic Pointing Modes, with axes and targets that are predefined according to the following table:

Table 4: Attitude control modes

Attitude	Primary	Primary Uffish thought	Secondary	Secondary Axis

Mode	Axis	Target	Axis	Target
YSM	-Z	Nadir	+X	EF Velociy
ZDOP	-Y	EF Velocity	-X	Nadir
LNP	-Z	Nadir	+X	Inertial Velocity
GP	Z	EF Satellite Position	+X	Inertial Velocity

The current Satellite Reference Frame (SRF) can be modeled in several ways by the user, for example:

- with three consecutive rotations of the SNRF over three user-defined angles (mispointing angles);
- with a user-provided misalignment matrix between SNRF and SRF;
- modulating the misalignment angles with biases and harmonics (provided by the user);
- reading attitude angles from a file.

8.2 Earth

8.2.1 Earth Position

The position and velocity of the Earth in the Barycentric and Heliocentric Mean of 2000 reference frames will be calculated according to FLANDERN reference.

8.2.2 Earth Geometry

The geometry of the Earth is modeled by a Reference Ellipsoid. Different definitions of reference ellipsoids can be found hereafter in Table 5 .

Table 5: WGS84 parameters

Parameter	Notation	WGS 84
Semi major axis (m)	a	6378137
Flattening = (a-b)/a	f	1/298.257223563
First Eccentricity = $(a^2 - b^2)/a^2$	e	0.0818191908426
Semi minor axis (m)	b	6356752.3142

The minimum distance between two points located on an ellipsoid is the length of the geodesic that crosses those two points. This geodesic distance will be calculated according to HEISKANEN reference.

The surface at a certain ***geodetic altitude h*** over the Earth's Reference Ellipsoid is defined by:

$$x = (N+h)\cos(\varphi)\cos(\lambda)$$

$$x = (N+h)\cos(\varphi)\sin(\lambda)$$

$$x = [(1-e^2)N + h]\sin(\varphi)$$

where N is the radius of curvature parallel to the meridian:

$$N = \frac{a}{\sqrt{1-e^2\sin^2\varphi}}$$

and φ and λ are the geodetic latitude and geocentric longitude of a point on that ellipsoid.

Nevertheless, the surface at a certain geodetic altitude h over the Earth´s Reference Ellipsoid will be modelled as another ellipsoid, *concentric* with it, and with (a+h) and (b+h) as semi-major and semi-minor axis.

This simplification is quite accurate and have the advantage that allows the analytical calculation of the intersection or tangent points with such a surface.

8.2.3 Earth Atmosphere

The Earth atmosphere can be represented by different models. The selection of a certain atmosphere model depends upon the requirements imposed by the mission definition. It could include certain simplifications to the generic definition.

It is also assumed that the atmosphere rotates with the same angular velocity as the Earth.

The definition of the Earth atmosphere is important for instrument pointing task and refraction.

8.2.3.1 US Standard Atmosphere 1976

The U.S Standard Atmosphere 1976 Atmosphere model is modified as follows:

- it ranges from Z = 0 Km to Z = 86 Km.
- the ratio M/M_0 decreases linearly from Z = 80 to Z = 86 Km.
- the linear relationship between T_M and H is replaced by either an arc of a circle or by a polynomial function in the vicinity of the points where the molecular-scale temperature gradient changes, in order to have a continuous and differentiable function $T_M = f(H)$

The U.S Standard Atmosphere 1976 is defined as follows (STD76 reference):

- The air is assumed to be dry, and at altitudes sufficiently below 86 Km, the atmosphere is assumed to be homogeneously mixed with a relative-volume composition leading to a constant mean molecular weight M.
- The air is treated as if it were a perfect gas, and the total pressure P, temperature T, and total density ρ at any point in the atmosphere are related by the equation of state, i.e. the perfect gas law, one form of which is:

$$P = \frac{\rho R T}{M}$$

where R = 8.31432 x 10^3 [Nm/(KmolK)] is the universal gas constant.

- Besides the atmosphere is assumed to be in hydrostatic equilibrium, and to be horizontally stratified so that dP, the differential of pressure, is related to dZ, the differential of geometric altitude, by the relationship:

$$dP = -g\,\rho\,dZ$$

where g is the altitude-dependent acceleration of gravity, which can be calculated with the expression:

$$g = g_0 \left(\frac{r_0}{r_0 + Z}\right)^2$$

where r_0 = 6356766 [m] and g_0 = 9.80665 [m/s^2], and that yields:

$$H = \frac{r_0 Z}{r_0 + Z}$$

where H is the geopotential altitude.

- The molecular-scale temperature T_M at a point is defined as:

$$T_M = T\,\frac{M_0}{M}$$

where M_0 = 28.9644 [Kg/Kmol] is the sea-level value of M.

In the region from Z = 0 Km to Z = 80 Km M is constant and M = M_0, whereas between Z = 80 Km and Z = 86 Km, the ratio M/M_0 is assumed to decrease from 1.000000 to 0.999578

Up to altitudes up to 86 Km the function T_M versus H is expressed as a series of seven successive linear equations. The general form of these linear equations is:

$$T_M = T_{M,b} + L_{M,b}(H - H_0)$$

The value of $T_{M,b}$ for the first layer (b = 0) is 288.15 [K], identical to T_0 the sea-level value of T.

The six values of H_b and $L_{M,b}$ are:

Table 6: Molecular-scale temperature coefficients

Subscript	Geopotential altitude H_b [Km]	Molecular-scale temperature gradient LM,b [K/Km]
0	0	-6.5
1	11	0.0
2	20	1.0
3	32	2.8
4	47	0.0

5	51	-2.8
6	71	-2.0
7	84.8520 (Z = 86)	

Finally, the pressure can be calculated with the following expressions:

$$P = P_b \left(\frac{T_{M,b}}{T_{M,b} + L_{M,b}(H - H_b)} \right)^{\frac{g_0 M_0}{R L_{M,b}}} \quad (L_{M,b} \neq 0)$$

$$P = P_b \exp\left[\frac{-g_0 M_0 (H - H_b)}{R\, T_{M,b}} \right] \quad (L_{M,b} = 0)$$

The reference-level value for P_b for b = 0 is the defined sea-level value P_0 = 101325.0 N/m^2. Values of P_b for b = 1 through b ≥ 6 are obtained from the application of the appropriate equation above for the case when H = H_{b+1}.

8.2.4 Refractive index

The refractive index is calculated with the Edlen's law (EDLEN) although neglecting the contribution of the partial pressure of the water vapor.

8.2.4.1 Edlen´s law

The relative refraction index m at any point in the atmosphere is calculated by:

$$m = 1 + N \times 10^{-6}$$

with
$$N = \left[a_0 + \frac{a_1}{1 - \left(\frac{\nu}{b_1}\right)^2} + \frac{a_2}{1 - \left(\frac{\nu}{b_2}\right)^2} \right] \frac{P}{P_0} \frac{(T_0 + 15{,}0)}{T} - \left[c_0 - \left(\frac{\nu}{c_1}\right)^2 \right] \frac{P_w}{P_0}$$

where P is the total air pressure in mb, T is the temperature in K, P_0 = 1013.25 mb, T_0 = 273.15 K, P_w is the partial pressure of water vapour in mb, and ν = $10^4/\lambda$ is the frequency in cm^{-1} for the wavelength λ in micrometers (EDLEN)

The constants in that equation are:

a_0 = 83.42

a_1 = 185.08

a_2 = 4.11

b_1 = 1.140 x 10^5

b_2 = 6.24 x 10^4

$c_0 = 43.49$

$c_1 = 1.70 \times 10^4$

The total air pressure and the temperature will be the corresponding to the atmosphere previously described, and the term in the last equation that corresponds to the partial pressure of water vapor will be neglected and therefore not calculated.

8.2.5 Digital Elevation Models (DEM)

The digital elevation model of the Earth consists in a set of points defining a grid for which a measure of the altitude over the Earth reference ellipsoid is given.

The following DEM types are supported:

- GETASSE30 (versions 1,2,3), see http://earth.esa.int/services/amorgos/download/getasse/
- ACE2, see http://tethys.eaprs.cse.dmu.ac.uk/ACE2/shared/overview
- ASTER GDEM, see http://asterweb.jpl.nasa.gov/gdem.asp

GETASSE30 and ACE2 DEM can be downloaded also from this URL:

http://eop-cfi.esa.int/index.php/mission-cfi-software/eocfi-software/support-files (registration required)

8.3 Sun and Moon

Sun and Moon position and velocity in the True of Date reference frame will be calculated according to FLANDERN reference.

8.4 Stars

To calculate the look direction from the satellite to a star, two consecutive steps must be performed:

- To calculate the stars coordinates in the Mean of Date reference frame at the current epoch, taking as input a star catalogue (assumed to be expressed in the Barycentric Mean of 2000.0 reference frame for the epoch J2000.0).
- To calculate the star coordinates in the Satellite Relative Actual Reference frame at the same epoch.

The first step must apply the following corrections:

Table 7: First step correction of star looking direction

Correction	Description	Effect

Proper motion	Intrinsic motion of the star across the background with respect to a reference epoch (e.g J2000.0) leading to a change in the apparent star position at the current epoch	Lower than 0.3 mdeg/year
Annual parallax	Apparent displacement of the position of the star caused by the difference in the position of the barycenter and the position of the Earth with the motion of the Earth around the Sun during the year	Lower than 0.3 mdeg
Light deflection	Gravitational lens effect of the Sun	Lower than 500 µdeg at the limb of the Sun and falling off rapidly with distance, e.g. to 6 µdeg at an elongation of 20 deg (so it will be ignored)
Annual aberration	Apparent displacement of the position of the star caused by the finite speed of light combined with the motion of the Earth around the Sun during the year	Lower than 6 mdeg
Precession	Change of the position of the star caused by the transformation from the Geocentric Mean of 2000.0 to the Mean of Date reference frame	Lower than 6.0 mdeg/year

whereas the second step must apply the following ones:

Table 8: Second step corrections of star looking direction

Correction	Description	Effect
Satellite parallax	Apparent displacement of the position of the star caused by the difference in the position of the satellite and the position of the Earth with the motion of the satellite around the Earth during an orbit	Lower than 0.015 µdeg even for the closest stars (so it will be ignored)
Satellite aberration	Apparent displacement of the position of the star caused by the finite speed of light combined with the motion of the satellite around the Earth during an orbit	Lower than 1 mdeg for LEO spacecraft

8.4.1 Stars Positions

To apply some of the necessary corrections to calculate the coordinates of a star in the Satellite Relative Actual Reference frame, the following expressions shall be used (ALMAN95 reference):

- Get the following variables from a star catalogue:

- Right ascension at J2000.0 expressed in the Barycentric Mean of 2000.0: α_0 [rad]
- Declination at J2000.0 expressed in the Barycentric Mean of 2000.0: δ_0 [rad]
- Proper motion in the right ascension: μ_α [rad/century]
- Proper motion in the declination: μ_δ [rad/century]
- Radial velocity: ν [au/century]
- Parallax: π [rad]

• Correct the star position obtained from the star catalogue (α_0, δ_0) for the proper motion and annual parallax effects using the expressions:

$$\bar{q} = (\cos\alpha_0\cos\delta_0, \sin\alpha_0\cos\delta_0, \sin\delta_0)$$

$$\bar{m} = (m_x, m_y, m_z)$$

$$m_x = -\mu_\alpha\cos\delta_0\sin\alpha_0 - \mu_\delta\sin\delta_0\cos\alpha_0 + \upsilon\pi\cos\delta_0\cos\alpha_0$$

$$m_y = \mu_\alpha\cos\delta_0\cos\alpha_0 - \mu_\delta\sin\delta_0\sin\alpha_0 + \upsilon\pi\cos\delta_0\sin\alpha_0$$

$$m_z = \mu_\delta\cos\delta_0 + \upsilon\pi\sin\delta_0$$

$$\bar{P} = \bar{q} + T\bar{m} - \pi\bar{r}_{B,\,Earth}$$

where T = (t - 0.5)/36525, and t is the current TDT expressed in the MJD2000 format, and $\vec{r}_{B,\,Earth}$ is the position of the Earth in AU at that TDT, expressed in the Barycentric Mean of 2000 reference frame.

• Correct the star position for the annual aberration effect, using the following expressions:

$$\bar{p}_2 = \frac{\dfrac{\bar{p}_1}{\beta} + \left(1 + \dfrac{\bar{p}_1 \bullet \bar{v}}{1 + \dfrac{1}{\beta}}\right)\bar{v}}{1 + \bar{p}_1 \bullet \bar{v}}$$

$$\bar{v} = \frac{\bar{v}_{B,\,Earth}}{c} = 0.0057755\,\bar{v}_{B,\,Earth}$$

$$\beta = \frac{1}{\sqrt{1 - |\bar{v}|^2}}$$

where $\vec{v}_{B,Earth}$ is the velocity of the Earth in AU/d at the current TDT expressed in the Barycentric Mean of 2000 reference frame.

• The satellite aberration can be calculated with the expression (IERS_SUPL reference):

$$\Delta\theta = \mathrm{asin}\left(\frac{v}{c}\sin\theta - \frac{1}{4}\left(\frac{v}{c}\right)^2\sin 2\theta\right) \quad \text{[rad]}$$

where $\Delta\theta$ is the change in the look direction from the satellite to the star, v is the velocity of the satellite expressed in the True of Date reference frame, and c is the velocity of the light in a vacuum.

- The following drawing sketches the satellite aberration:

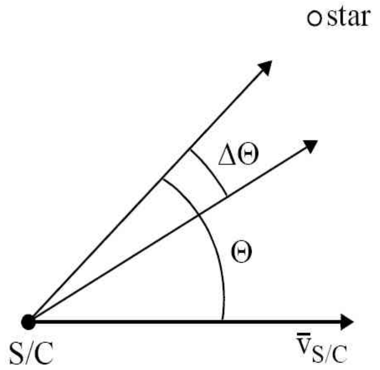


Figure 17: Satellite aberration

8.5 Ray tracing

The path followed by the light when it crosses the Earth´s atmosphere is bent due to the effect of the atmospheric refraction, and therefore the direction of the light when it enters the atmosphere $\vec{u}_E$ is different to the direction of the light when it leaves the atmosphere $\vec{u}_L$.

This effect is depicted in the following drawing:

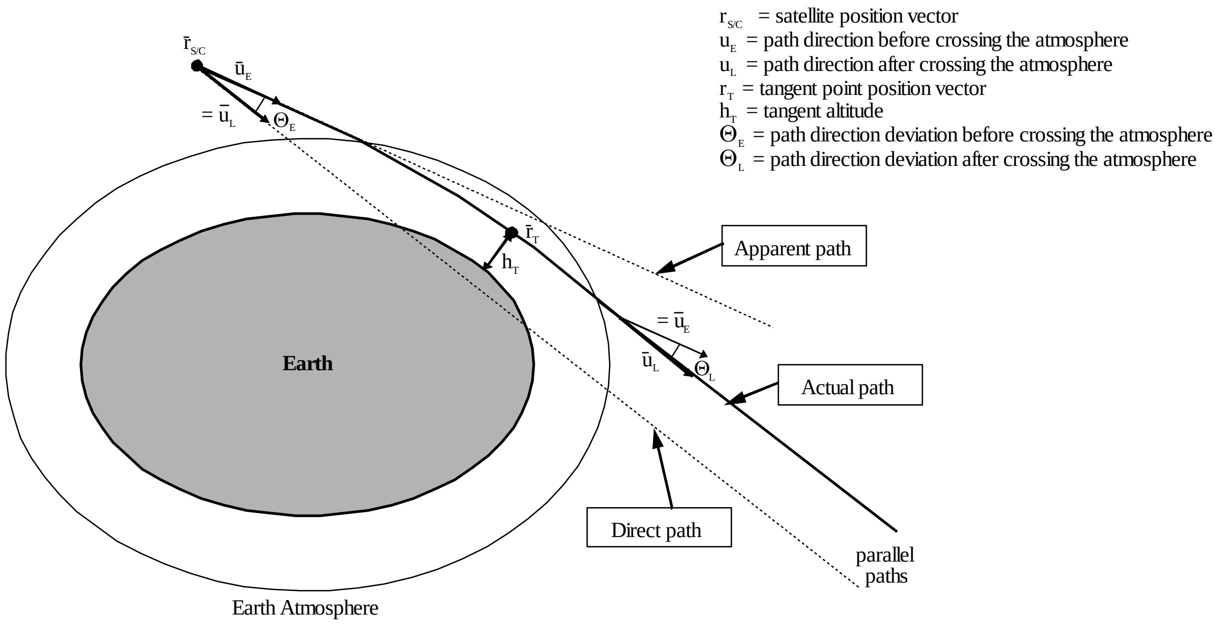


Figure 18: Ray path

Note that this drawing also represents the satellite position $\vec{r}_{SC}$, the tangent point of the light path over the Earth's Reference Ellipsoid $\vec{r}_T$ and the corresponding tangent altitude h_T, the maximum angular deviation of the light path at the entrance θ_E and at the exit θ_L of the atmosphere, where:

$$\cos\theta_E = \cos\theta_L = \vec{u}_E \cdot \vec{u}_L$$

The geometric distance $R_{T,g}$ between the satellite position and the tangent point is defined as:

$$R_{T,g} = |\vec{r}_T - \vec{r}_{SC}|$$

Finally, the range S_T is the actual distance between those two points measured along the path.

A **ray tracing model** calculates the path followed by the light from the satellite through the atmosphere to the observation target.

8.5.1 No refraction model

This is the simplest model as the effect of the atmospheric refraction is not considered, therefore the path followed by the light is approximated by a straight line.

The advantage of this model is that is purely analytical.

8.5.2 Refraction models

There are two refraction ray tracing models, both based on the three following assumptions:

- The relative refractive index is a function only of the geometric altitude H above the Earth´s Reference Ellipsoid, i.e. the Earth's atmosphere model is based on the assumption that the longitude or latitude variations of the relative refraction index are negligible
- The surface at a certain geometric altitude H will be modeled as an ellipsoid, *concentric* with the Earth's Reference Ellipsoid, with (a+H) and (b+H) as semi-major and semi-minor axis.
- The light path lays in a plane: this plane is defined by the satellite position vector $\vec{r}_{SC}$, and by the known light direction, either $\vec{u}_E$ or $\vec{u}_L$. This assumption implies that the three dimensional effects of the light path bending are assumed to be negligible.

This simplification is quite accurate and has the advantage to allow the analytical calculation of the intersection of tangent points with such a surface.

The light path is calculated by integrating the differential Eikonal´s equation in that plane:

$$\frac{d}{ds}\left(n\frac{d}{ds}\vec{r}\right)=\nabla m$$

where $\vec{r}$ is the position vector of a point in the light path, s is the arc length along that path, and m is the relative refraction index.

Note that iterative methods are usually needed to implement a refraction ray tracing model.

8.5.2.1 Standard atmosphere model

This refraction ray tracing model is based on the atmosphere model described in section 8.2.3.

8.5.2.2 User's atmosphere model

In this case the atmosphere model is based on a file supplied by the user, which defines the relative refraction index m at a discrete set of geometric altitudes H.

To have a continuous, and differentiable, relationship between the relative refraction index and the geometric altitude, the relative refraction index at any geometric altitude will be calculated by means of a cubic spline based on the two closest pair of data supplied by the user.

8.5.3 Predefined refraction corrective functions model

This model also assumes that the light path lays in a plane, i.e. the *reference* plane defined by the satellite position vector $\vec{r}_{SC}$, and by the known light direction, either $\vec{u}_E$ or $\vec{u}_L$.

It is based on the calculation of the parameters h_T, $R_{T,g}$, and S_T of the tangent point using the no refraction ray tracing model, and the calculation of a set of refraction corrective terms by means of predefined curves that depend only on the tangent altitude h_T and the wavelength of the light signal λ.

Those corrective terms are:

$$\Delta h_T=f_1(h_T,\lambda)$$

$$\Delta R_{T,g}=f_2(h_T,\lambda)$$

$$\Delta \theta_E=\Delta \theta_L=f_3(h_t,\lambda)$$

$$\Delta S_T=f_4(h_T,\lambda)$$

The tangent point, corrected for the effects of the atmosphere refraction, is calculated knowing that it lays in the *reference* plane, that the tangent altitude is $h_T + \Delta h_T$, and that the geometric distance from the satellite to the tangent point is $R_{T,g} + \Delta R_{T,g}$

The actual length of the light path is calculated as $S_T + \Delta S_T$

The light direction at the entrance or at the exit of the atmosphere, is calculated knowing that it lays in that reference plane, and that is deviated by an angle $\Delta\theta_E = \Delta\theta_L$.

The great advantage of this ray tracing model is that it is relatively quite accurate and is much faster than the *refraction* ray tracing models as it can be calculated analytically.

9 UNITS

In general, the units that will be used in all the CFI software will be the SI units, except for the angle that will use the ***degree*** instead of the ***radian.***

Table 9: Units in CFI Software

Quantity	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Thermodynamic temperature	kelvin	K
Amount of substance	mole	mol
Plane angle	degree	deg
Frequency	hertz	Hz
Pressure	pascal	Pa